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RESEARCH ARTICLE

Transboundary pollution control in asymmetric countries: do assistant investments help?

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Abstract
Transboundary pollution control usually requires the cooperation of neighboring countries due to the externality of pollution. However, countries at different levels of development, which are called asymmetric countries in this paper, may have different attitudes toward this cooperation. The developing countries would like to take a free ride and they can benefit from the cooperation with developed countries, but the developed countries may not be willing to afford this cooperation cost. This paper discusses the cooperation between two asymmetric countries that developed country may provide assistant investments to help the developing country reduce pollution stock. We consider a dynamic differential game to model the transboundary pollution control between two asymmetric regions and derive the optimal equilibrium of both regions using the Hamilton–Jacobi–Bellman (HJB) equation. To explore the impact of assistant investments, numerical illustrations and sensitivity analysis are implemented to compare the equilibrium strategies under two scenarios: that with or without assistant investments. We conclude that the common pollution stock will be reduced when the developed country is willing to provide assistant investments to the developing country. Besides, the equilibrium emission strategies of both countries increase with assistant investments, which means more economic benefits for both sides.

Keywords Transboundary pollution · Assistant investments · Dynamic differential game · Hamilton–Jacobi–Bellman equation

Introduction

In recent years, pollution from developing countries has become a threat to developed countries. For example, with the growth of economic activity in cities along the US–Mexico border, wastewater flows with gravity from the south in Mexico to the north in the US in the Tijuana watershed (Fernandez 2009). In March 2021, Mongolia was hit by a massive sandstorm. The dust storm then swept across much of northern China and caused severe dust and smog in South Korea. In fact, the source of air pollution in Northeast Asia has long been a matter of debate (Shapiro and Bolsen 2018). Due to the development of the domestic economy and the transfer of pollution-intensive industries from developed economies, developing countries are going through a phase of industrialization with relatively heavy pollution. It is an inevitable price in the process of economic development (Taylor 2005). The developing countries are faced with the dual task of economic development and environmental protection.

Advanced green or clean technology can help to reduce pollution and avoid repeating the past path of developed countries, which caused much pollution around the world (Anderson 1996; Goldemberg 1998; Yacob et al. 2019). However, advanced green technology usually requires a large amount of investments and developing countries cannot afford it (Avedissian 2002; Hasper 2007; Hasan et al. 2019). Developed countries have advantages in terms of capital and research and development capabilities. They may consider making assistant investments for developing countries to reduce the common pollution stock. Actually, international cooperation has become common sense in the transboundary

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pollution control problem. Based on the principle that "common but differentiated responsibilities" formulated by the United Nations Framework Convention on Climate Change in 1992, assistant investments in green technology and compulsory patent licensing for developing countries became possible (Brunnee and Streck 2013). The Cancun Climate Change Conference (COP16) in 2010 proposed to establish a "Green Climate Fund," which is planned to reach \$100 billion annually by 2020 to help poorer countries reduce emissions, partly by funding green technology investments (Fridahl and Linnér 2016). The Paris Climate Change Conference (COP21) in December 2015 called for the cooperation between developing and developed countries to improve their investment in research and development of green technologies (Biancardi and Villani 2018). International environmental laws have recognized the importance of technology assistance by creating a series of law mechanisms (Alam 2020).

However, the problem is that assistant investments from developed countries to developing countries are never pervasive despite all these international conventions. Most of the existing researches believed that international technical assistance benefits the developing countries only and the developed countries do not have enough motivation to implement assistant investments. In this paper, we consider the transboundary pollution control in asymmetric countries and discuss whether assistant investments from developed countries to developing countries help to reduce the common pollution stock and improve the economic performance of both sides. A dynamic differential game model in a finite horizon is constructed to characterize the evolution of pollution stock. Through the comparison of two scenarios, that with or without assistant investments, we aim to answer the following research questions:

1. What are the optimal strategies for both countries to balance economic development and pollution control?
2. How do assistant investments affect the emission levels and pollution stock?
3. Do assistant investments help to control transboundary pollution?

We aim to prove that the green technology assistance between asymmetric countries is a win-win strategy for both sides through mathematical models. Our researches provide theoretical support for the cooperation and promote assistant investments in transboundary pollution.

This paper is organized as follows. "Literature reviews" presents literature reviews. "Model" introduces a dynamic differential game model to characterize the transboundary pollution control, and "Two scenarios" discusses the optimal strategies under two scenarios. "Numerical simulations" implements numeric illustrations to demonstrate our conclusions. "Conclusions" briefly concludes this paper.

Literature reviews

Most researchers believe that international technical assistance is an appropriate tool to support the development of poor areas (Easterly and Williamson 2011). Not only in the economic area, but the environment also improved. Niho (1996) proposed that international assistant investments can help improve the environment by providing advanced green technology to developing countries. Haibara (2002) pointed out that technical assistance is better than direct financial assistance in terms of pollution abatement. Arvin et al. (2009) made an econometric analysis of the relationship between assistant investments and economic development level. They found that these aids improve the economic development level of recipient countries significantly. Park (2016) discussed the management of environmental official development assistance (ODA) to improve the quality of life through environmental protection in poorer countries. Simone and Bazilian (2019) presented a real example that international agencies provided electricity services in sub-Saharan Africa, which reduced the air pollution from fuels for cooking and lighting.

The externality of pollution makes the transboundary pollution problem more difficult. Adaptation and mitigation policies are important issues in the study of transboundary pollution. Ingham et al. (2007) proposed a two-phase transboundary pollution game for adaptation and mitigation strategies with uncertainty and learning. Then Ingham et al. (2013) studied whether adaptation and mitigation are complements or not. Brechet et al. (2013) discussed the optimal allocation of investments in mitigating the harm of climate change and environmental adaptation from the perspective of the macroeconomy. They found that countries with higher economic efficiency had a lower optimal policy mix for adaptation and mitigation. Yatsenko et al. (2014) later made a specific analysis to show that the adaptive long-term investment is profitable only when the economy exceeds a certain efficiency threshold. Brechet et al. (2016) analyzed the strategic behavior of several countries engaged in capital accumulation, pollution mitigation, and environmental adaptation in the context of the environmental common good, and they proposed an effective international environmental cooperation agreement. Lazkano et al. (2016) and Breton and Sbragia (2017, 2019) also analyzed mitigation and adaptation policies in transboundary pollution issues from the perspective of international environmental agreements.

Early empirical models on transboundary pollution mainly focus on the impact of a particular policy on a particular country over a particular period (Kaitala et al. 1992a, 1992b). In recent decades, the dynamic game models in multiple countries have been emerging as a promising approach. Asymmetric scenarios, including economic, geographical, and ideological asymmetries, were considered in the differential game models of transboundary pollution (Ludema and

Wooton 1997). Jørgensen and Zaccour (2001) established a dynamic game of upstream and downstream transboundary pollution from the perspective of environmental asymmetry and proved that cooperative solutions were better than non-cooperative solutions by using the intertemporal decomposition method. Biancardi and Villani (2014) proposed international environmental agreements with developed and developing countries in a dynamic approach. Chang et al. (2018) obtained optimal emission levels and abatement expenditures in a finite horizon transboundary pollution game with emission trading between two regions. Other related works include Masoudi and Zaccour (2018) and Xu and Tan (2019).

Model

In this paper, we consider two neighboring and asymmetric countries, i.e., the developed and developing countries, in a finite horizon T. Let $i \in \{1, 2\}$ denote the developed and developing countries, respectively.

At each moment $t \in [0, T]$, the economic benefits of each country $R_i(t)$ depends on the industrial production, which is positively related to its emissions $E_i(t)$. Jørgensen et al. (2010) and Vardar and Zaccour (2018) assumed a linear–quadratic function between economic benefits and emissions.

$$R_i(t) = a_i E_i(t) - \frac{b_i}{2} E_i^2(t), \tag{1}$$

where a_i, $b_i > 0$ are corresponding coefficients. Haibara (2002) and Gallup and Marcotte (2004) pointed out that technical assistance can help to reduce pollution and improve productivity. Here, we introduce an additional productive efficiency coefficient α for developing countries, i.e.,

$$\begin{cases} R_1(t) = a_1 E_1(t) - \dfrac{b_1}{2} E_1^2(t), \\ R_2(t) = (a_2 + \alpha) E_2(t) - \dfrac{b_2}{2} E_2^2(t), \end{cases} \tag{2}$$

where $\alpha > 0$ when developing countries receive assistant investments from developed countries and otherwise $\alpha = 0$.

The emissions of industrial production are accumulated as pollution stock in the region. Since the two countries are neighboring, they are assumed to suffer from the common pollution stock $P(t)$. Benchekroun and Taherkhani (2014) proposed that pollution stock is a linear form associated with emissions. In this paper, we also consider the effect of assistant investments $I(t)$ in reducing pollution. Then the common pollution stock of this region evolves according to the following dynamics:

$$\begin{cases} \dfrac{dP(t)}{dt} = E_1(t) + E_2(t) - \phi I(t) - \delta P(t), \\ P(0) = P_0, \end{cases} \tag{3}$$

where $\phi > 0$ is the efficiency of investments for pollution control and $\delta > 0$ is a natural degradation rate of pollution. If the developed country declines to support the developing country, the assistant investments $I(t) = 0$. Such assistant investments take costs of manpower and material resources. The cost function of the developed country is defined as

$$C(t) = \frac{\varphi}{2} I^2(t), \tag{4}$$

where $\varphi > 0$ is the corresponding coefficient. Equation 4 implies that the costs of investments are quadratically increasing, which is a common setting in economic researches.

The accumulated pollution stock causes environmental damage in each country. Long (1992), Dockner and Van Long (1993), and Benchekroun et al. (2017) proposed the environmental damage is a quadratic function of pollution stock:

$$D_i(t) = \frac{\omega_i}{2} P^2(t), \tag{5}$$

where $\omega_i > 0$ are the environmental damage coefficients.

Based on the aforementioned functions, the pay-off of the two countries can be defined as follows:

$$\begin{aligned} \Pi_1 &= \int_0^T e^{-\rho t} \{R_1(t) - D_1(t) - C(t)\} dt + e^{-\rho T} U_1(P(T)), \\ \Pi_2 &= \int_0^T e^{-\rho t} \{R_2(t) - D_2(t)\} dt + e^{-\rho T} U_2(P(T)), \end{aligned} \tag{6}$$

where ρ is a discount rate and $U_1(P(T))$ and $U_2(P(T))$ are the salvage values of two countries after a finite horizon T. Assuming that the salvage values can be well approximated by linear functions:

$$U_i(P(T)) = \lambda_i (Q - P(T)), \tag{7}$$

where Q is the upper limit of environmental tolerable pollution and $\lambda_i > 0$ are corresponding coefficients.

So far, we have defined a two-player dynamic differential game model in a finite horizon. Both countries would maximize their own pay-off functions with control variables, i.e., $\{E_1(t), I(t)\}$ for the developed country and $\{E_2(t)\}$ for the developing country, under the common state variable $P(t)$ defined in Eq. 3.

Two scenarios

In this section, we derive the optimal equilibrium solutions of the aforementioned differential game. The equilibrium solutions in the two scenarios that the developed country provides assistant investments to the developing country or not are compared to explore the impact of assistant investments. Here, the dynamic differential game is played in a finite horizon T, which is a more realistic setting due to the rapid changes in policy and technology. The planning cycle can be divided into small intervals, such as 1 year (annual plan) or one quarter. However, the differential game in a finite horizon is more difficult to solve than that in an infinite horizon since the resulting Ricatti equations form a system of differential equations, rather than algebraic equations. The equilibrium solutions can only be obtained numerically.

With assistant investments

If the developed country is willing to provide assistant investments to the developing country, i.e., $I(t) > 0$, both countries maximize their own pay-off function with control variables.

$$
\begin{aligned}
&{}^{max}\prod\nolimits_1^{E_1(t),I(t)} = \max_{E_1(t),\,I(t)} \int_0^T e^{-\rho t}\{R_1(t)-D_1(t)-C(t)\}dt + e^{-\rho T}U_1(P(T)), \\
&{}^{max}\prod\nolimits_2^{E_2(t)} = \max_{E_2(t)} \int_0^T e^{-\rho t}\{R_2(t)-D_2(t)\}dt + e^{-\rho T}U_2(P(T))
\end{aligned}
\tag{8}
$$

We solve this optimization according to the Hamilton–Jacobi–Bellman (HJB) equation (Dockner et al. 2000). The HJB function associated with such optimal control problem in region i can be denoted by $V_i^{(1)}(t, P(t))$, $i = 1, 2$:

$$
\begin{aligned}
&\rho V_1^{(1)}(t,P)-\frac{\partial V_1^{(1)}}{\partial t} \\
&= \overline{\max} E_1(t), I(t)\left\{R_1(t)-D_1(t)-C(t) + \frac{\partial V_1^{(1)}}{\partial P}(E_1(t)+E_2(t)-\phi I(t)-\delta P(t))\right\} \\
&= \overline{\max} E_1(t), I(t)\left\{a_1E_1(t)-\frac{b_1}{2}E_1^2(t)-\frac{\omega_1}{2}P^2(t)-\frac{\varphi}{2}I^2(t) + \frac{\partial V_1^{(1)}}{\partial P}(E_1(t)+E_2(t)-\phi I(t)-\delta P(t))\right\}
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&\rho V_2^{(1)}(t,P)-\frac{\partial V_2^{(1)}}{\partial t} \\
&= \overline{\max} E_2(t)\left\{R_2(t)-D_2(t) + \frac{\partial V_2^{(1)}}{\partial P}(E_1(t)+E_2(t)-\phi I(t)-\delta P(t))\right\} \\
&\overline{\max} E_2(t)\left\{(a_2+\alpha)E_2(t)-\frac{b_2}{2}E_2^2(t)-\frac{\omega_2}{2}P^2(t) + \frac{\partial V_2^{(1)}}{\partial P}(E_1(t)+E_2(t)-\phi I(t)-\delta P(t))\right\}
\end{aligned}
\tag{10}
$$

where $V_1^{(1)}(T, P(T)) = U_1(P(T))$ and $V_2^{(1)}(T, P(T)) = U_2(P(T))$.

By taking derivation of the right-hand side of Eqs. 9 and 10 with respect to their control variables, we obtain the following system and solve it:

$$
\begin{cases}
a_1-b_1E_1(t)+\dfrac{\partial V_1^{(1)}}{\partial P}=0, \\
-\varphi I(t)-\phi\dfrac{\partial V_1^{(1)}}{\partial P}=0 \\
a_2+\alpha-b_2E_2(t)+\dfrac{\partial V_2^{(1)}}{\partial P}=0
\end{cases}
=>
\begin{cases}
\widehat{E}_1(t)=\dfrac{a_1}{b_1}+\dfrac{1}{b_1}\dfrac{\partial V_1^{(1)}}{\partial P}, \\
\widehat{I}(t)=-\dfrac{\phi}{\varphi}\dfrac{\partial V_1^{(1)}}{\partial P}, \\
\widehat{E}_2(t)=\dfrac{a_2+\alpha}{b_2}+\dfrac{1}{b_2}\dfrac{\partial V_2^{(1)}}{\partial P}
\end{cases}
\tag{11}
$$

The following proposition presents the characterization of the feedback Nash equilibrium in this scenario:

- Proposition 1:

If there exist value functions $V_i^{(1)}(t, P(t))$, $i = 1, 2$ and the feedback strategy $\left\{\widehat{I}(t), \widehat{E}_1(t), \widehat{E}_2(t)\right\}$ satisfies the equilibrium conditions given in Eq. 11 for all $t \in [0, T]$, then the strategy constitutes a feedback Nash equilibrium. Moreover, the value function $V_i^{(1)}(t, P(t))$ represents the equilibrium total payoff of the ith country with assistant investments.

By substituting the equilibrium strategy given in Eq. 11 into Eqs. 9 and 10, we have

$$\begin{aligned}\rho V_1^{(1)}(t,P(t)) - \frac{\partial V_1^{(1)}}{\partial t} = \beta_1 + \beta_2 P^2(t) + \beta_3 \frac{\partial V_1^{(1)}}{\partial P} \\ + \beta_4 \frac{\partial V_1^{(1)}}{\partial P} P(t) \\ + \beta_5 \left(\frac{\partial V_1^{(1)}}{\partial P}\right)^2 \\ + 2\beta_6 \frac{\partial V_1^{(1)}}{\partial P} \frac{\partial V_2^{(1)}}{\partial P},\end{aligned} \tag{12}$$

and

$$\begin{aligned}\rho V_2^{(1)}(t,P(t)) - \frac{\partial V_2^{(1)}}{\partial t} = \beta_7 + \beta_8 P^2(t) + \beta_3 \frac{\partial V_2^{(1)}}{\partial P} \\ + \beta_4 \frac{\partial V_2^{(1)}}{\partial P} P(t) \\ + \beta_6 \left(\frac{\partial V_2^{(1)}}{\partial P}\right)^2 \\ + 2\beta_5 \frac{\partial V_1^{(1)}}{\partial P} \frac{\partial V_2^{(1)}}{\partial P},\end{aligned} \tag{13}$$

where $\beta_1 = \frac{{a_1}^2}{2b_1}$, $\beta_2 = -\frac{\omega_1}{2}$, $\beta_3 = \frac{a_1}{b_1} + \frac{a_2+\alpha}{b_2}$, $\beta_4 = -\delta$, $\beta_5 = \frac{1}{2b_1} + \frac{\phi^2}{2\varphi}$, $\beta_6 = \frac{1}{2b_2}$, $\beta_7 = \frac{(a_2+\alpha)^2}{2b_2}$, and $\beta_8 = -\frac{\omega_2}{2}$.

Note that $V_1^{(1)}(t,P(t))$ and $V_2^{(1)}(t,P(t))$ follow linear–quadratic structures. Let

$$\begin{aligned}V_1^{(1)}(t,P(t)) = f_1(t) + f_2(t)P(t) + f_3(t)P^2(t), \\ V_2^{(1)}(t,P(t)) = f_4(t) + f_5(t)P(t) + f_6(t)P^2(t),\end{aligned} \tag{14}$$

where $f_1(t)$, ……, $f_6(t)$ are coefficients only depending on time t. Then by substituting Eq. 14 into Eqs. 12 and 13 and comparing both sides of equations for constant order, $P(t)$-order, and $P^2(t)$-order, respectively, we have the following six-dimensional Ricatti differential equations system (Haurie et al. 2012):

$$\begin{aligned}\rho f_1(t) - f_1'(t) = \beta_1 + \beta_3 f_2(t) + \beta_5 f_2^2(t) + 2\beta_6 f_2(t) f_5(t), \\ \rho f_2(t) - f_2'(t) = 2\beta_3 f_3(t) + \beta_4 f_2(t) + 4\beta_5 f_2(t) f_3(t) + 4\beta_6 (f_2(t) f_6(t) + f_3(t) f_5(t)), \\ \rho f_3(t) - f_3'(t) = \beta_2 + 2\beta_4 f_3(t) + 4\beta_5 f_3^2(t) + 8\beta_6 f_3(t) f_6(t), \\ \rho f_4(t) - f_4'(t) = \beta_7 + \beta_3 f_5(t) + \beta_6 f_5^2(t) + 2\beta_5 f_2(t) f_5(t), \\ \rho f_5(t) - f_5'(t) = 2\beta_3 f_6(t) + \beta_4 f_5(t) + 4\beta_6 f_5(t) f_6(t) + 4\beta_5 (f_2(t) f_6(t) + f_3(t) f_5(t)), \\ \rho f_6(t) - f_6'(t) = \beta_8 + 2\beta_4 f_6(t) + 4\beta_6 f_6^2(t) + 8\beta_5 f_3(t) f_6(t)\end{aligned} \tag{15}$$

It is hard to obtain an analytic solution to such a differential equation system. We solve this differential equation system with the salvage value function in Eq. 7 numerically in "Numerical simulations." To satisfy the conditions of $U1(P(T))$ and $U2(P(T))$, we have:

$$\begin{aligned}f_1(T) = \lambda_1 Q, f_2(T) = -\lambda_1, f_3(T) = 0, \\ f_4(T) = \lambda_2 Q, f_5(T) = -\lambda_2, f_6(T) = 0.\end{aligned} \tag{16}$$

For the concerned control variables of both countries, by substituting Eq. 14 into Eq. 11, we have

$$\begin{aligned}\widehat{E}_1(t,P(t)) = \frac{a_1}{b_1} + \frac{1}{b_1} f_2(t) + \frac{2}{b_1} f_3(t) P(t), \\ \widehat{E}_2(t,P(t)) = \frac{a_2+\alpha}{b_2} + \frac{1}{b_2} f_5(t) + \frac{2}{b_2} f_6(t) P(t), \\ \widehat{I}(t,P(t)) = -\frac{\phi}{\varphi} f_2(t) - \frac{2\phi}{\varphi} f_3(t) P(t).\end{aligned} \tag{17}$$

Equation 17 implies that the feedback equilibrium strategies depend on both time t and the state variable $P(t)$. Numerical simulations are implemented in "Numerical simulations" to analyze the strategies of both countries and the equilibrium trajectory of pollution stock.

Without assistant investments

If the developed country rejects to provide assistant investments to the developing country, i.e., $I(t) = 0$, both countries maximize their own pay-off function with control variables.

$$\begin{aligned}{}^{max}\prod\nolimits_1^{E_1(t)} = \frac{\max}{E_1(t)} \int_0^T e^{-\rho t}\{R_1(t) - D_1(t)\}dt + e^{-\rho T} U_1(P(T)), \\ {}^{max}\prod\nolimits_2^{E_2(t)} = \frac{\max}{E_2(t)} \int_0^T e^{-\rho t}\{R_2(t) - D_2(t)\}dt + e^{-\rho T} U_2(P(T)).\end{aligned} \tag{18}$$

Similarly, the HJB function associated with such optimal control problem in region i can be denoted by

$$\begin{aligned}\rho V_1^{(2)}(t,P) - \frac{\partial V_1^{(2)}}{\partial t} \\ = \frac{\max}{} E_1(t) \left\{ R_1(t) - D_1(t) + \frac{\partial V_1^{(2)}}{\partial P} (E_1(t) + E_2(t) - \delta P(t)) \right\} \\ = \frac{\max}{} E_1(t) \left\{ a_1 E_1(t) - \frac{b_1}{2} E_1^2(t) - \frac{\omega_1}{2} P^2(t) + \frac{\partial V_1^{(2)}}{\partial P} (E_1(t) + E_2(t) - \delta P(t)) \right\}\end{aligned} \tag{19}$$

$$\begin{aligned}\rho V_2^{(2)}(t,P) - \frac{\partial V_2^{(2)}}{\partial t} \\ = \frac{\max}{} E_2(t) \left\{ R_2(t) - D_2(t) + \frac{\partial V_2^{(2)}}{\partial P} (E_1(t) + E_2(t) - \delta P(t)) \right\} \\ = \frac{\max}{} E_2(t) \left\{ a_2 E_2(t) - \frac{b_2}{2} E_2^2(t) - \frac{\omega_2}{2} P^2(t) + \frac{\partial V_2^{(2)}}{\partial P} (E_1(t) + E_2(t) - \delta P(t)) \right\}\end{aligned} \tag{20}$$

where $V_1^{(2)}(T,P(T)) = U_1(P(T))$ and $V_2^{(2)}(T,P(T)) = U_2(P(T))$.

By taking derivation of the right-hand side of Eqs. 19 and 20 with respect to their control variables, we obtain the following system and solve it:

$$\begin{cases} a_1 - b_1 E_1(t) + \dfrac{\partial V_1^{(2)}}{\partial P} = 0, \\ a_2 - b_2 E_2(t) + \dfrac{\partial V_2^{(2)}}{\partial P} = 0, \end{cases} => \begin{cases} \widetilde{\mathrm{E}}_1(\mathrm{t}) = \dfrac{\mathrm{a}_1}{\mathrm{b}_1} + \dfrac{1}{\mathrm{b}_1} \dfrac{\partial \mathrm{V}_1^{(2)}}{\partial \mathrm{P}}, \\ \widetilde{\mathrm{E}}_2(\mathrm{t}) = \dfrac{\mathrm{a}_2}{\mathrm{b}_2} + \dfrac{1}{\mathrm{b}_2} \dfrac{\partial \mathrm{V}_2^{(2)}}{\partial \mathrm{P}}. \end{cases} \tag{21}$$

The following proposition presents the characterization of the feedback Nash equilibrium in this scenario:

- Proposition 2:

If there exist value functions $V_i^{(2)}(t, P(t)), i = 1, 2$ and the feedback strategy $\left\{\widetilde{\mathrm{E}}_1(\mathrm{t}), \widetilde{\mathrm{E}}_2(\mathrm{t})\right\}$ satisfies the equilibrium conditions given in Eq. 21 for all $t \in [0, T]$, then the strategy constitutes a feedback Nash equilibrium. Moreover, the value function $V_i^{(2)}(t, P(t))$ represents the equilibrium total payoff of the ith country with assistant investments.

By substituting the equilibrium strategy given in Eq. 21 into Eqs. 19 and 20, we have

$$\begin{aligned} \rho V_1^{(2)}(t, P(t)) - \frac{\partial V_1^{(2)}}{\partial t} = \gamma_1 + \gamma_2 P^2(t) + \gamma_3 \frac{\partial V_1^{(2)}}{\partial P} \\ + \gamma_4 \frac{\partial V_1^{(2)}}{\partial P} P(t) \\ + \gamma_5 \left(\frac{\partial V_1^{(2)}}{\partial P} \right)^2 \\ + 2\gamma_6 \frac{\partial V_1^{(2)}}{\partial P} \frac{\partial V_2^{(2)}}{\partial P}, \end{aligned} \tag{22}$$

and

$$\begin{aligned} \rho V_2^{(2)}(t, P(t)) - \frac{\partial V_2^{(2)}}{\partial t} = \gamma_7 + \gamma_8 P^2(t) + \gamma_3 \frac{\partial V_2^{(2)}}{\partial P} \\ + \gamma_4 \frac{\partial V_2^{(2)}}{\partial P} P(t) \\ + \gamma_6 \left(\frac{\partial V_2^{(2)}}{\partial P} \right)^2 \\ + 2\gamma_5 \frac{\partial V_1^{(2)}}{\partial P} \frac{\partial V_2^{(2)}}{\partial P}, \end{aligned} \tag{23}$$

where $\gamma_1 = \frac{{a_1}^2}{2b_1}, \gamma_2 = -\frac{\omega_1}{2}, \gamma_3 = \frac{a_1}{b_1} + \frac{a_2}{b_2}, \gamma_4 = -\delta, \gamma_5 = \frac{1}{2b_1}, \gamma_6 = \frac{1}{2b_2}, \gamma_7 = \frac{{a_2}^2}{2b_2}$, and $\gamma_8 = -\frac{\omega_2}{2}$.

Note that $V_1^{(2)}(t, P(t))$ and $V_2^{(2)}(t, P(t))$ follow linear–quadratic structures. Let

$$\begin{aligned} V_1^{(2)}(t, P(t)) = g_1(t) + g_2(t)P(t) + g_3(t)P^2(t), \\ V_2^{(2)}(t, P(t)) = g_4(t) + g_5(t)P(t) + g_6(t)P^2(t), \end{aligned} \tag{24}$$

where $g_1(t)$, ……, $g_6(t)$ are coefficients only depending on time t. Then by substituting Eq. 24 into Eqs. 22 and 23 and comparing both sides of equations for constant order, $P(t)$-order, and $P^2(t)$-order, respectively, we have the following six-dimensional Ricatti differential equations system:

$$\begin{aligned} \rho g_1(t) - g_1'(t) = \gamma_1 + \gamma_3 g_2(t) + \gamma_5 g_2^2(t) + 2\gamma_6 g_2(t) g_5(t), \\ \rho g_2(t) - g_2'(t) = 2\gamma_3 g_3(t) + \gamma_4 g_2(t) + 4\gamma_5 g_2(t) g_3(t) + 4\gamma_6 (g_2(t) g_6(t) + g_3(t) g_5(t)), \\ \rho g_3(t) - g_3'(t) = \gamma_2 + 2\gamma_4 g_3(t) + 4\gamma_5 g_3^2(t) + 8\gamma_6 g_3(t) g_6(t), \\ \rho g_4(t) - g_4'(t) = \gamma_7 + \gamma_3 g_5(t) + \gamma_6 g_5^2(t) + 2\gamma_5 g_2(t) g_5(t), \\ \rho g_5(t) - g_5'(t) = 2\gamma_3 g_6(t) + \gamma_4 g_5(t) + 4\gamma_6 g_5(t) g_6(t) + 4\gamma_5 (g_2(t) g_6(t) + g_3(t) g_5(t)), \\ \rho g_6(t) - g_6'(\mathrm{t}) = \gamma_8 + 2\gamma_4 g_6(t) + 4\gamma_6 g_6^2(t) + 8\gamma_5 g_3(t) g_6(t), \end{aligned} \tag{25}$$

Similarly, we solve this differential equation system with the salvage value function in Eq. 7 numerically in "Numerical simulations." To satisfy the conditions of $U1(P(T))$ and $U2(P(T))$, we have

$$\begin{aligned} g_1(T) = \lambda_1 Q, g_2(T) = -\lambda_1, g_3(T) = 0, \\ g_4(T) = \lambda_2 Q, g_5(T) = -\lambda_2, g_6(T) = 0 \end{aligned} \tag{26}$$

For the concerned control variables of both countries, by substituting Eq. 24 into Eq. 21, we have

$$\begin{aligned} \widetilde{E}_1(t, P(t)) = \frac{a_1}{b_1} + \frac{1}{b_1} g_2(t) + \frac{2}{b_1} g_3(t) P(t), \\ \widetilde{E}_2(t, P(t)) = \frac{a_2}{b_2} + \frac{1}{b_2} g_5(t) + \frac{2}{b_2} g_6(t) P(t). \end{aligned} \tag{27}$$

Equation 27 implies that the feedback equilibrium strategies depend on both time t and the state variable $P(t)$. Numerical simulations are implemented in "Numerical simulations" to analyze the strategies of both countries and the equilibrium trajectory of pollution stock.

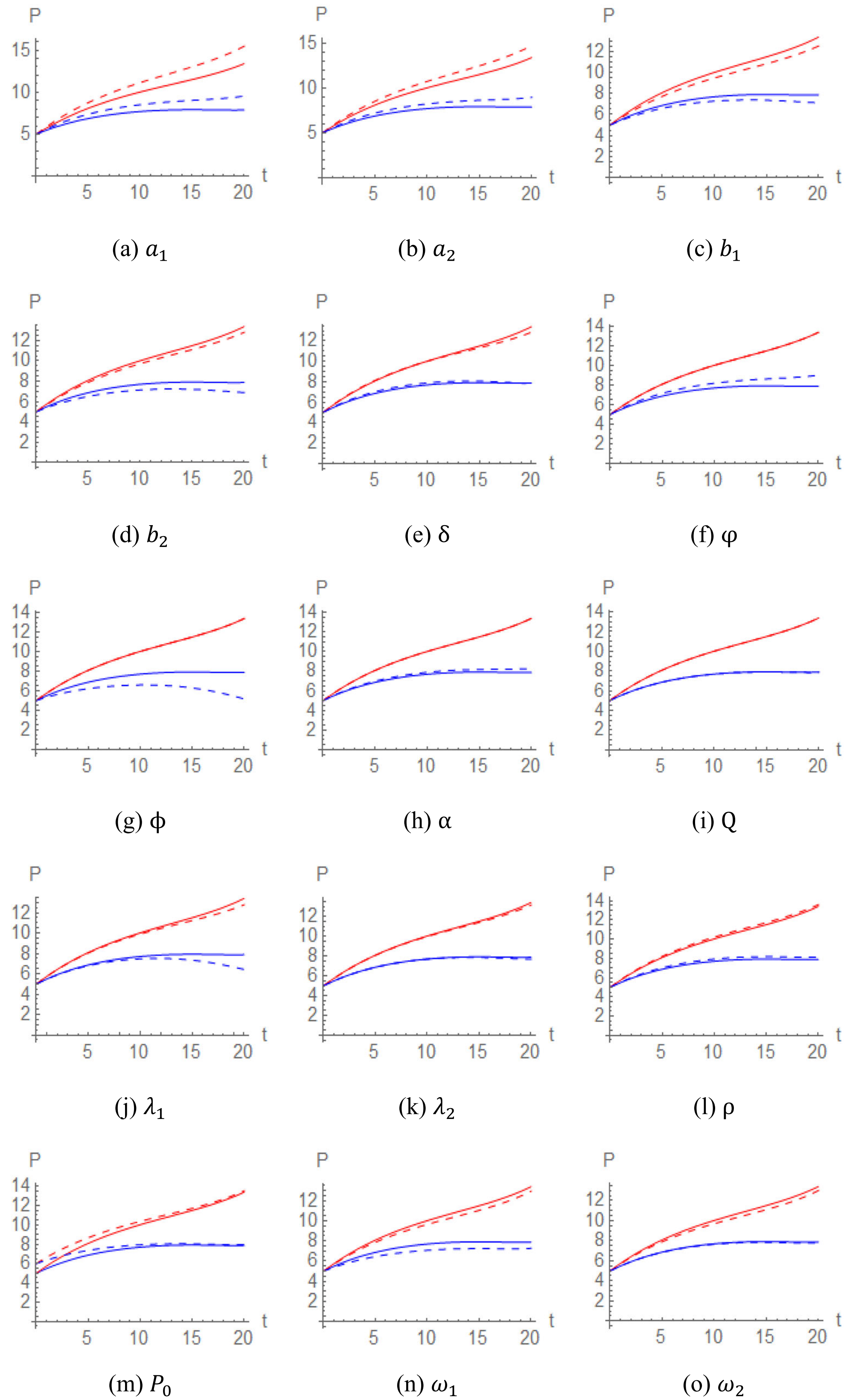
Numerical simulations

In this section, we implement numerical simulations to explore the equilibrium strategies for each country. By comparing the results in two scenarios, the effects of the assistant investments on the pollution stock and emission strategies of each region are illustrated.

Table 1 Input data of parameters

Parameter	a_1	a_2	b_1	b_2	Q	α	ϕ	φ
Value	1	0.8	0.8	1	200	0.3	1.3	0.8
Parameter	ρ	ω_1	ω_2	δ	T	λ_1	λ_2	P_0
Value	0.025	0.005	0.006	0.05	20	0.5	0.25	5

Fig. 1 Legends: — denotes the trajectory of pollution stock under the benchmark setting with assistant investments; — denotes the trajectory of pollution stock under the benchmark setting without assistant investments; - - - denotes the trajectory of pollution stock with assistant investments when each parameter is increased by 20%; - - - denotes the trajectory of pollution stock without assistant investments when each parameter is increased by 20%



The Ricatti equations in Eqs. 15 and 25 are two-point boundary-value problems, which can be solved using Mathematica's numerical differential equation solver (NDSolve Functionality). Then, according to the solutions of

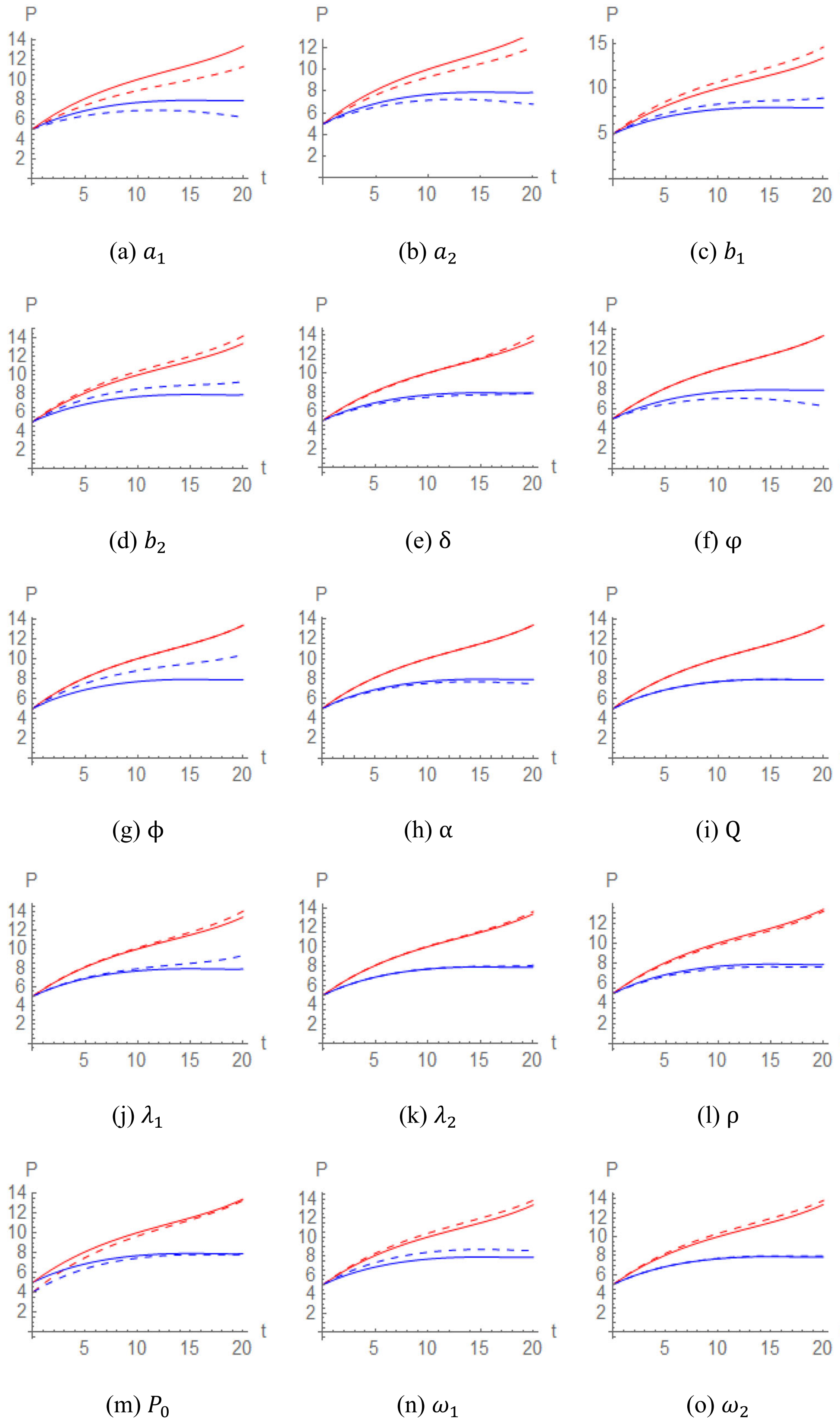


Fig. 2 Legends: — denotes the trajectory of pollution stock under the benchmark setting with assistant investments; — denotes the trajectory of pollution stock under the benchmark setting without assistant investments; - - - denotes the trajectory of pollution stock with assistant investments when each parameter is decreased by 20%; - - - denotes the trajectory of pollution stock without assistant investments when each parameter is decreased by 20%

Ricatti equations, we can obtain the optimal equilibrium strategies $\left\{\widehat{E}_1(t), \widehat{I}(t), \widehat{E}_2(t)\right\}$ by Eq. 11 and $\left\{\widetilde{E}_1(t), \widetilde{E}_2(t)\right\}$ by Eq. 21. Specifically, the parameters in the model are assigned in Table 1 as an example. Note that the parameter setting in Table 1 is just an example to show the trends and effects of results. Here, we focus on the ordinal relation of different parameters rather than specific values. For example, since the developed country will gain more economic benefits from the same amount of emission, we have $a_1 > a_2$ and $b_1 < b_2$. We also perform sensitivity analysis for the values of parameters in the following section. It draws similar conclusions under all parameter settings.

The trajectory of pollution stock over time t is denoted by $\widehat{P}(t)$ if the developed country is willing to provide assistant investments and is denoted by $\widetilde{P}(t)$ if not. Based on the benchmark settings in Table 1, we implement the sensitivity analysis by increasing and decreasing each parameter by 20% individually. Figures 1 and 2 present the trajectory of pollution stock in each setting. They demonstrate that the pollution stock is lower when the developed country is willing to provide assistant investments to the developing country in all parameter settings, i.e.,

$$\widehat{P}(t) < \widetilde{P}(t)$$

for all $t \in [0, T]$. This conclusion is reasonable since the assistant investments from the developed country always help to reduce the accumulation of pollution stock.

Figure 3 presents the trajectory of equilibrium emission strategies over time t under both scenarios based on the benchmark setting. It illustrates that both countries increase their emission when the developed country is willing to provide assistant investments to the developing country, i.e.,

$$\widehat{E}_1(t) > \widetilde{E}_1(t), \widehat{E}_2(t) > \widetilde{E}_2(t)$$

for all $t \in [0, T]$. It shows a quite positive result that both countries generate more emissions, but the common pollution stock is reduced due to the assistant investments.

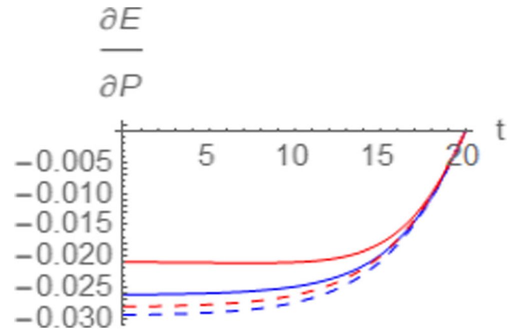


Fig. 4 Legends: — denotes the gradient of equilibrium emission strategies for the developed country with assistant investments; - - - denotes the gradient of equilibrium emission strategies for the developed country without assistant investments; — denotes the gradient of equilibrium emission strategies for the developing country with assistant investments; - - - denotes the gradient of equilibrium emission strategies for the developing country without assistant investments

Figure 4 presents the gradient of $E(t)$ with respect to $P(t)$ over t. The equilibrium emission strategies have a negative relationship with the pollution stock, i.e.,

$$\frac{\partial \widehat{E}_i(t, P(t))}{\partial P} < 0, \frac{\partial \widetilde{E}_i(t, P(t))}{\partial P} < 0$$

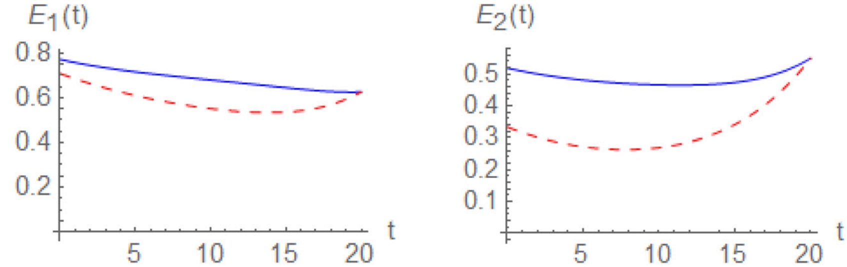
for $i \in \{1, 2\}$ and all $t \in [0, T]$. It is easy to understand since there exists an upper limit of environmental tolerable pollution at time T to restrict the emission. When the time t is close to the terminal T, the gradient converges to zero.

Figure 5 presents the gradient of $I(t)$ with respect to $P(t)$ over t. The equilibrium assistant investments have a positive relationship with the pollution stock, i.e.,

$$\frac{\partial I(t, P(t))}{\partial P} > 0.$$

It demonstrates that the developing country needs more assistant investments if the pollution stock is higher. Also, the gradient converges to zero when the time is close to the terminal.

Fig. 3 Legends: — denotes the trajectory of equilibrium emission strategies with assistant investments; - - - denotes the trajectory of equilibrium emission strategies without assistant investments



(a) $E_1(t)$ (b) $E_2(t)$

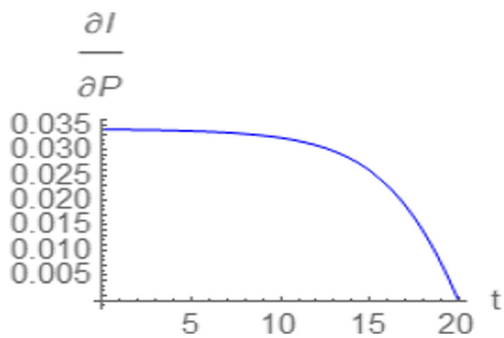


Fig. 5 The trajectory of the gradient of equilibrium assistant investments

Conclusions

The main objective of this paper discusses whether the assistant investments from the developed country to the developing country help to reduce the pollution stock or not. We consider a dynamic differential game to model the transboundary pollution control between two asymmetric regions and derive the optimal equilibrium of both regions using the HJB equation. Different from most existing researches, which only focus on the benefit of developing regions through assistant investments, this paper demonstrates that both developed and developing countries benefit from the cooperation by using a differential game model.

Through the previous analysis, we can draw the following conclusions. The common pollution stock will be reduced if the developed country is willing to provide assistant investments to the developing country. At the same time, the equilibrium emission strategies of both countries increase and so they may produce more economic benefits with the assistant investments. It is a win-win game for both sides. Under a finite horizon plan, the effect of emission strategies and the level of investments converge to zero when the time is close to the terminal.

Besides the dynamic differential game, other models for transboundary pollution control will be considered in the future to compare with our results.

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Declarations

Ethics approval and consent to participate Not applicable.

Consent for publication Not applicable.

Competing interests The authors declare no competing interests.

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