Science of the Total Environment 852 (2022) 158286

Contents lists available at ScienceDirect

Science of the Total Environment

journal homepage: www.elsevier.com/locate/scitotenv

Payments for environmental services strategy for transboundary air pollution: A stochastic differential game perspective

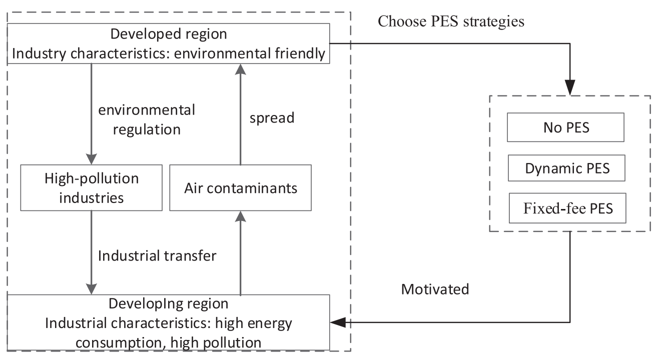
Jianyue Liu [a], Lu Xiao [a], Jun Wang [b], Chaojie Wang [c,*]

[a] School of Management, Jiangsu University, Zhenjiang 212013, China
[b] Institute of Applied System Analysis, Jiangsu University, Zhenjiang 212013, China
[c] School of Mathematical Science, Jiangsu University, Zhenjiang 212013, China

HIGHLIGHTS

- The transboundary air pollution control problem between two asymmetric regions within the JPCAP area is assessed.
- A stochastic differential game model is proposed to model the diffusion of air pollution.
- Both PES strategies (dynamic and fixed-fee payment) decrease the amounts of air pollutants.
- The dynamic PES strategy has better performance than the fixed-fee PES strategy

GRAPHICAL ABSTRACT



ARTICLE INFO

Editor: Pavlos Kassomenos

Keywords:
Payments for environmental services
Stochastic differential game
Transboundary air pollution
Hamilton-Jacobi-Bellman equation

ABSTRACT

Air pollution has become a global threat to societal development. The main challenges of transboundary air pollution control include two perspectives: uneven socioeconomic development of regions and the diffusion of air pollution. This paper proposes an PES strategy to alleviate transboundary air pollution by coordinating regional economic interests and environmental preferences within the joint prevention and control of air pollution region. To make the model design more realistic, we introduce the stochastic differential game model to characterize the diffusion and uncertainty of air pollution. The optimal feedback Nash equilibrium is derived in three PES scenarios (no PES, dynamic PES, and fixed-fee PES) by using the Hamilton-Jacobi-Bellman equation. Numerical simulations and sensitivity analysis are implemented to compare the optimal strategies under the three PES scenarios. The dynamic PES strategy is shown to outperform the no PES strategy and the fixed-fee PES strategy by encouraging the backward region to cut more emissions. Besides, the confidence interval theory is used to estimate the variation range of air pollution stocks, which provides a powerful diagnostic tool for policy-makers.

1. Introduction

With the increase of greenhouse gas emissions due to human activity and natural change (Angelevska et al., 2021; Feistel and Hellmuth, 2021), air pollution has become a global threat to societal development (Jacobson, 2009). Some studies demonstrated that more than three million people died from illnesses caused by prolonged exposure to air pollution each year (Dockery et al., 1993; Zhang et al., 2017), and the death toll may double by 2050 (Lelieveld et al., 2015). Recently, more and more countries have realized the urgency of air pollution control. Over the past decade, the annual investment in environmental pollution control has

* Corresponding author.
E-mail address: cjwang@ujs.edu.cn (C. Wang).

http://dx.doi.org/10.1016/j.scitotenv.2022.158286
Received 17 June 2022; Received in revised form 19 August 2022; Accepted 22 August 2022
Available online 31 August 2022

occupied 10 % of the gross domestic product (GDP) in China (Liu et al., 2017). However, traditional strategies for air pollution control are inefficient due to the incompatibility between the transregional spillover of air pollutants and the single-jurisdiction policy based on administrative division (Qin et al., 2015). Hence, the joint prevention and control of air pollution (JPCAP) has become a global consensus to solve severe transnational and transregional air pollution problems (Wang and Zhao, 2018). The South Coast Air Quality Management District (SCAQMD) in the USA and the Beijing-Tianjin-Hebei (BTH) region in China are both standard examples of JPCAP (Kamieniecki and Ferrall, 1991; Xiao et al., 2020).

The establishment of JPCAP helps to strengthen cross-regional cooperation, alleviate externalities of pollution and improve the efficiency of air pollution control (Xie et al., 2018). However, in practice, fiscal decentralization leads to “local development-oriented governments”. Within the JPCAP region, uneven regional socioeconomic development (such as economic development levels, energy and industrial structures, technology, and population) results in the phenomena of “freeriding” (Bina, 2010). For an example of the BTH region, Beijing and Tianjin, which have higher economic development levels, tend to put more emphasis on air quality. But economic development is still the primary goal for Hebei province, even if it also benefits from air pollution control. The motivation for cutting capacity to reduce emissions in Hebei province is always insufficient. The members of the JPCAP region eventually fall into the dilemma of coordination.

To avoid the dilemma mentioned above, the PES mechanism has attracted increasing attention. PES is defined as voluntary transactions between service users and service providers that are conditional on agreed rules of natural resource management for generating offsite services (Wunder, 2015). It is based on the principle of beneficiary-pay rather than polluter-pay, and thus is attractive in settings where environmental services providers are poor, marginalized landholders or powerful groups of actors (Engel et al., 2008). By connecting the beneficiaries and providers of environmental services, PES has become the core content of transboundary environmental pollution control (Yang et al., 2013; Kolinjivadi et al., 2014; Lyle et al., 2015). Through the PES mechanism, the externality of environmental pollution can be quantified into financial incentives for each member. Eventually, it may improve the efficiency of transregional cooperation and hence reduce pollution. Examples include the Pago por Servicios Ambientales (PSA) program in Costa Rica (Pagiola, 2008), the Payment for Hydrological Environmental Services (PSAH) program in Mexico (Muñoz-Piña et al., 2008) and the agri-environmental payment program in the USA (Claassen et al., 2008). In 2016, the State Council of China released a national plan on environmental improvements during the 13th Five-Year Plan period (2016–2020). It proposed accelerating the establishment of a diversified PES mechanism to improve the efficiency of air pollution control in the BTH region.

Many studies have been conducted in order to improve the efficiency of PES (Roumasset and Wada, 2013; Mahanty et al., 2013; Xiong et al., 2017). Sun et al. (2017) considered the conservation cost method, the market value method, and the payment ability method to determine payment standards for the Middle Route Project of the South-to-North Water Diversion Project in China. Do et al. (2018) combined online surveys and in-depth interviews to explore the motivations of private-sector buyers for environmental services in Vietnam, and put forward relevant policy suggestions that the government should amend laws and regulatory procedures to make environmental services more marketable. Recently, differential game theory has been an important tool to model PES strategies by characterizing the accumulation of pollutant stocks over time and the game between participants. Jørgensen and Zaccour (2001) studied the cross-border pollution problems of neighboring countries based on the differential game model. They designed an intertemporal decomposition scheme for the total side payment. Andrés-Domenech et al. (2015) employed an empirical differential game to explore the optimal total amount of PES that the non-forest owners have to transfer to forest owners in order to ensure sustainable forest management. Jiang et al. (2019) applied a differential game model to characterize the transboundary pollution control covering upstream and downstream areas, and obtained the optimal ecological compensation and cooperation strategy between regions.

In this paper, we focus on the transboundary air pollution control problem between two asymmetric members within the JPCAP region. The main challenges of transboundary air pollution control include two perspectives: uneven socioeconomic development of regions and the diffusion of air pollution. The PES strategy provides an effective approach to balance regional economic interests and environmental preferences within the JPCAP region. In contrast to the previous studies, this paper specifies and compares three different payment strategies between the developed and developing regions, including no PES, dynamic PES, and fixed-fee PES. No PES means the scenario without payments for environmental services. Dynamic PES refers to the strategy where payments from the developed region depend on the actual amounts of reduced emissions in the developing region. Fixed-fee PES is a one-off payment strategy that depends on whether the developing region agrees to join in the cooperation. We derive the optimal feedback Nash equilibrium in three PES scenarios by using the Hamilton-Jacobi-Bellman (HJB) equation. The dynamic PES strategy is shown to outperform the no PES strategy and the fixed-fee PES strategy by encouraging the backward region to cut more emissions.

Compared with recent advances in applying the PES strategy to watersheds, farmland, and forest preservation, another challenge for transboundary air pollution control is to model the diffusion of air pollution, which is highly stochastic and is influenced by many variables such as weather conditions (Wang et al., 2019). Moreover, industrial environmental policies may be uncertain due to changes in demographic structures and economic statistics (Huang et al., 2006). To characterize the uncertainty in the real world, we introduce the stochastic differential game (SDG) model to the transboundary air pollution control problem. The SDG is an emerging and promising aspect in the field of differential game theory, which makes the model design more realistic. Yeung (2007) pioneered applying the stochastic differential game theory to transboundary industrial pollution management between industries and governments. Chang et al. (2015) proposed a stochastic differential game to simulate the cross-border industrial pollution problems under emission permit trading. They indicated that the stochastic emission permits prices can motivate the players to make more flexible decisions in the games. Yi et al. (2017) constructed a cooperative stochastic differential game of transboundary industrial pollution between two asymmetric nations at an infinite-horizon level and proposed a payment distribution mechanism that supports the subgame consistent solution. In this paper, we consider the SDG model under the PES strategy for transboundary air pollution control. To the best of our knowledge, this is the first time the SDG model and the PES strategy have been combined to address the transboundary air pollution control problem.

In general, this paper contributes to the following three perspectives: First, we explore three PES strategies for transboundary air pollution control between two asymmetric members within the JPCAP region. We derive the optimal feedback Nash equilibrium in three PES scenarios by using the HJB equation. Numerical simulations and sensitivity analysis are implemented to compare the optimal strategies under the three PES scenarios. The dynamic PES strategy is shown to outperform the no PES strategy and the fixed-fee PES strategy by encouraging the backward region to cut more emissions. Second, we combine the SDG model and the PES strategy to characterize the uncertainty of air pollution. To the best of our knowledge, this is the first time the SDG model and the PES strategy have been combined to address the transboundary air pollution control problem. Third, different from previous studies that provided point estimation of environmental pollution based on data-driven models and trend analysis methods (Ekwueme and Agunwamba, 2021; Zhao et al., 2021), this paper uses the confidence interval theory to estimate the variation range of air pollution stocks, which provides a more powerful diagnostic tool for policymakers.

This paper is organized as follows: Section 2 introduces the SDG model for the transboundary air pollution problem. Section 3 considers the three scenarios of PES respectively. Section 4 presents the numerical illustrations of the results under equilibrium states in different PES scenarios and sensitivity analysis of parameters. Section 5 summarizes the paper briefly and suggests future research directions. The proofs of our results are included in the Appendix.

2. SDG model

Production activities bring economic utility $R_h(t)$ along with air pollutant emissions $e_h(t)$, where the developed region and the developing region are indexed by $h = i, j$, respectively. Time t runs continuously and the planning horizon is infinite. The relationship between utility and emissions can be modeled as follows (Breton et al., 2010; Li, 2014):

$$R_h(t) = A_h e_h(t) - \frac{1}{2} e_h^2(t), \tag{2.1}$$

where A_h is a parameter to denote the economic and technological levels of two regions. Let $A_i = A$ and $A_j = \theta A$, where $A > 0$ and $\theta \in (0,1)$ (Chang et al., 2018). Then we have $A_i > A_j$ to characterize the difference in economic levels. Eq. (2.1) illustrates that utility is a linear-quadratic function of emissions, which conforms to the law of diminishing marginal utility.

Note that $e_h(t)$ denotes the gross air pollutant emissions in region h. For the reasons of environment protection, the region may reduce a certain proportion of emissions $\alpha_h(t)$ at the cost $C_h(t)$ (Ye and Zhao, 2016),

$$C_h(t) = \frac{1}{2} c_h [\alpha_h(t) e_h(t)]^2, \tag{2.2}$$

where c_h is the unit cost of emission reduction. In general, region h reduces $\alpha_h(t)e_h(t)$ emissions by taking the cost $C_h(t)$ and remains the rest $(1 - \alpha_h(t))e_h(t)$ emissions. To describe the asymmetry of economic and technological levels between two regions, we set $c_i = c - \delta$ and $c_j = c$, where $c > 0$ and $\delta \in (0, c)$ (Wang, 1998). Here, c denotes the unit cost of reduction in developing region. The developed region with advanced technology has a lower unit cost denoted by $c - \delta$.

Nakada et al. (2013) pointed out that each region has an upper limit of tolerance of air pollutants $\bar{e}_h$, which depends on the economic and environmental level. Air quality can be expressed as the difference between $\bar{e}_h$ and local amounts of air pollutant emissions. Different from De Frutos and Martín-Herrán (2019), which only considered one-way spillover of pollution stock, this paper considers the interdiffusion of air pollutants between two regions. Let ω_i be the proportion of transmitted emissions from region i to j, and ω_j is vice versa. Thus, air quality of two regions $Q_h(t)$ can be expressed as follows:

$$\begin{cases} Q_i(t) = \bar{e}_i - (1-\omega_i)(1-\alpha_i(t))e_i(t) - \omega_j\left(1-\alpha_j(t)\right)e_j(t), \\ Q_j(t) = \bar{e}_j - \left(1-\omega_j\right)\left(1-\alpha_j(t)\right)e_j(t) - \omega_i(1-\alpha_i(t))e_i(t). \end{cases} \tag{2.3}$$

Then the utility function of air quality in region h, Z_h, can be written as $Z_h(t) = \beta_h Q_h(t)$, where β_h is the utility coefficient of air qualities in region h. $Z_h(t)$ denotes the potential benefits generated from good environments.

Practically, due to the existence of the "air basin", air pollutants emitted from two regions will accumulate in the restricted areas (Qin et al., 2015). So we can assume that there are no interactions between the two regions and outside regions. Besides, the accumulation of air pollutants might be influenced by natural degradation and other unpredictable variables (Athanassoglou and Xepapadeas, 2012; Masoudi et al., 2016). To characterize the dynamic process of gross air pollutants in both regions $P(t)$, we proposed a stochastic differential equation as follows:

$$\begin{cases} dP(t) = \left\{(1-\alpha_i(t))e_i(t) + \left(1-\alpha_j(t)\right)e_j(t) - \phi P(t)\right\}dt + \sigma(P(t))dr(t), \\ P(0) = P_0 \geq 0, \end{cases} \tag{2.4}$$

where $r(t)$ is a standard Brownian motion and $\sigma(P(t))$ is a stochastic volatility term. This functional form is a variation of the standard stochastic differential equation (Iacus, 2008) and implies the uncertainty of air pollutants in the real world. For simplicity, we assume that $\sigma(P(t)) = \sigma\sqrt{P(t)}$, where $\sigma > 0$ is a constant. At each moment of time, the gross air pollution stock $P(t)$ will increase by the total of emissions from both regions. Meanwhile, it will decrease at the natural degradation rate denoted by $\phi \in (0,1)$. The initial value is set as $P(0) = P_0$.

Then the damage costs, also known as environmental degradation costs, caused by air pollution can be written as (Chang et al., 2018):

$$D_h(t) = k_h P(t), \tag{2.5}$$

where k_h reflects the potential economic loss due to the unit air pollutant in region h. The pollution in the developed region may cause more economic loss than the developing region. So let $k_i = k$ and $k_j = \mu k$, where $k > 0$ and $\mu \in (0,1)$. It reflects the asymmetric economic development level between the two regions.

3. PES strategies

Eq. (2.5) illustrates that the developed region has a stronger motivation to reduce pollution since it costs more potential economic loss than the developing region. Therefore, the developed region may consider paying certain fees to help the developing region reduce air pollutant emissions. In this section, we discuss PES strategies under the three scenarios. Specifically, the superscript $s \in \{N, D, F\}$ denotes the scenarios without PES, dynamic PES, and fixed-fee PES, respectively. In all these scenarios, both regions play a Stackelberg non-cooperative game using a feedback information structure.

3.1. Equilibrium under scenario N

Under this scenario, there is no environmental cooperation between the two regions. The developed region pays no compensation to the developing region. They maximize their own gross discounted instantaneous payoffs, respectively, as follows:

$$\begin{aligned} \Pi_i^N &= \max_{e_i(t),\alpha_i(t)} \int_0^\infty e^{-\rho t}\{R_i(t) - C_i(t) - D_i(t) + Z_i(t)\}dt \\ &= \max_{e_i(t),\alpha_i(t)} \int_0^\infty e^{-\rho t}\Big\{A_i e_i(t) - \frac{1}{2}e_i^2(t) - \frac{1}{2}c_i[\alpha_i(t)e_i(t)]^2 - k_i P(t) \\ &\quad + \beta_i\left[\bar{e}_i - (1-\omega_i)(1-\alpha_i(t))e_i(t) - \omega_j\left(1-\alpha_j(t)\right)e_j(t)\right]\Big\}dt, \end{aligned} \tag{3.1}$$

$$\begin{aligned} \Pi_j^N &= \max_{e_j(t),\alpha_j(t)} \int_0^\infty e^{-\rho t}\left\{R_j(t) - C_j(t) - D_j(t) + Z_j(t)\right\}dt \\ &= \max_{e_j(t),\alpha_j(t)} \int_0^\infty e^{-\rho t}\Big\{A_j e_j(t) - \frac{1}{2}e_j^2(t) - \frac{1}{2}c_j\left[\alpha_j(t)e_j(t)\right]^2 - k_j P(t) \\ &\quad + \beta_j\left[\bar{e}_j - \left(1-\omega_j\right)\left(1-\alpha_j(t)\right)e_j(t) - \omega_i(1-\alpha_i(t))e_i(t)\right]\Big\}dt, \end{aligned} \tag{3.2}$$

where $\rho \in (0,1)$ is a discount rate over time. Specifically, this game consists of two control variables for each region, e_h, α_h, and one state variable, $P(t)$, which follows a stochastic differential equation in Eq. (2.4). Thus, it is a stochastic differential game problem.

We solve this problem according to the HJB equation. The HJB equation is a result of dynamic programming theory, pioneered by Richard Bellman in the 1950s (Bellman, 1954). Recently, it has been widely used and become a standard solution for stochastic control problems (Kemajou-Brown, 2016). The HJB function associated with such an optimal control problem in region h can be denoted by $V_h^N(P)$,

$$\begin{aligned}\rho V_i^N(P) = \max_{e_i(t),\alpha_i(t)} & A_i e_i(t) - \frac{1}{2}e_i^2(t) - \frac{1}{2}c_i[\alpha_i(t)e_i(t)]^2 - k_i P(t) \\ & + \beta_i\left[\bar{e}_i - (1-\omega_i)(1-\alpha_i(t))e_i(t) - \omega_j\left(1-\alpha_j(t)\right)e_j(t)\right] \\ & + V_i^{N'}(P)\left\{[1-\alpha_i(t)]e_i(t) + \left[1-\alpha_j(t)\right]e_j(t) - \phi P(t)\right\} \\ & + V_i^{N''}(P)\frac{1}{2}(\sigma(P(t)))^2,\end{aligned} \tag{3.3}$$

$$\begin{aligned}\rho V_j^N(P) = \max_{e_j(t),\alpha_j(t)} & A_j e_j(t) - \frac{1}{2}e_j^2(t) - \frac{1}{2}c_j\left[\alpha_j(t)e_j(t)\right]^2 - k_j P(t) \\ & + \beta_j\left[\bar{e}_j - \left(1-\omega_j\right)\left(1-\alpha_j(t)\right)e_j(t) - \omega_i(1-\alpha_i(t))e_i(t)\right] \\ & + V_j^{N'}(P)\left\{[1-\alpha_i(t)]e_i(t) + \left[1-\alpha_j(t)\right]e_j(t) - \phi P(t)\right\} \\ & + V_j^{N''}(P)\frac{1}{2}(\sigma(P(t)))^2,\end{aligned} \tag{3.4}$$

where $V_h^{N'}(P) = \frac{dV_h^N(P)}{dP}$ and $V_h^{N''}(P) = \frac{d^2V_h^N(P)}{dP^2}$.

By solving above HJB equations, the optimal feedback strategies for two regions can be obtained in Proposition 1.

Proposition 1. *Under the scenario N, the Markov-perfect Nash equilibrium solutions of the instantaneous emissions strategies $e_h(t)$, the proportion of emission reduction $\alpha_h(t)$, and the total payoff of region h, $h = i, j$ can be written as follows*:

$$\begin{aligned}
e_i^{N*} &= A_i - \beta_i(1-\omega_i) - \frac{k_i}{\phi+\rho}, \quad e_j^{N*} = A_j - \beta_j\left(1-\omega_j\right) - \frac{k_j}{\phi+\rho}, \\
\alpha_i^{N*} &= \frac{\beta_i(1-\omega_i)(\phi+\rho)+k_i}{c_i[A_i(\phi+\rho) - \beta_i(1-\omega_i)(\phi+\rho) - k_i]}, \\
\alpha_j^{N*} &= \frac{\beta_j\left(1-\omega_j\right)(\phi+\rho)+k_j}{c_j\left[A_j(\phi+\rho) - \beta_j\left(1-\omega_j\right)(\phi+\rho) - k_j\right]}, \\
V_i^{N*} &= -\frac{k_i}{\phi+\rho}P(t) + \frac{1}{\rho}\left\{\left(1+\frac{1}{c_j}\right)\frac{k_i k_j}{(\phi+\rho)^2} + \frac{1}{2}\left(1+\frac{1}{c_i}\right)\frac{k_i^2}{(\phi+\rho)^2} + \beta_i\omega_j\left(1+\frac{1}{c_j}\right)\frac{k_j}{\phi+\rho}\right. \\
&\quad - \left\{A_i + A_j - \beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right) - \beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)\right\}\frac{k_i}{\phi+\rho} \\
&\quad + \frac{1}{2}\beta_i^2(1-\omega_i)^2\left(1+\frac{1}{c_i}\right) + \beta_i(\bar{e}_i - A_i(1-\omega_i) - A_j\omega_j) \\
&\quad \left. + \frac{1}{2}A_i^2 + \left(1+\frac{1}{c_j}\right)\beta_i\beta_j\omega_j(1-\omega_j)\right\}, \\
V_j^{N*} &= -\frac{k_j}{\phi+\rho}P(t) + \frac{1}{\rho}\left\{\left(1+\frac{1}{c_i}\right)\frac{k_i k_j}{(\phi+\rho)^2} + \frac{1}{2}\left(1+\frac{1}{c_j}\right)\frac{k_j^2}{(\phi+\rho)^2} + \beta_j\omega_i\left(1+\frac{1}{c_i}\right)\frac{k_i}{\phi+\rho}\right. \\
&\quad - \left\{A_i + A_j - \beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right) - \beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)\right\}\frac{k_j}{\phi+\rho} \\
&\quad \left. + \frac{1}{2}\beta_j^2(1-\omega_j)^2\left(1+\frac{1}{c_j}\right) + \beta_j(\bar{e}_j - A_j(1-\omega_j) - A_i\omega_i) + \frac{1}{2}A_j^2 + \left(1+\frac{1}{c_i}\right)\beta_i\beta_j\omega_i(1-\omega_i)\right\}.
\end{aligned} \tag{3.5}$$

Proofs of propositions see Appendix A for details.

To describe the evolution path of amounts of air pollutants, we calculate the expectation and variance of $P^N(t)$ under equilibrium states in Proposition 2.

Proposition 2. *Under the scenario N, the expectation and variance of amounts of air pollutants $P^N(t)$ satisfy*:

$$\begin{cases} E\left(P^N(t)\right) = \dfrac{\Omega^N}{\phi} + \left(P_0 - \dfrac{\Omega^N}{\phi}\right)e^{-\phi t}, \\ S\left(P^N(t)\right) = \dfrac{\sigma^2\left\{\Omega^N - 2\left[\Omega^N - \phi P_0\right]e^{-t\phi} + \left[\Omega^N - 2\phi P_0\right]e^{-2\phi t}\right\}}{2\phi^2}, \end{cases} \tag{3.6}$$

where $\Omega^N = A_i + A_j - (1+\frac{1}{c_i})(\beta_i(1-\omega_i) + \frac{k_i}{\phi+\rho}) - (1+\frac{1}{c_j})(\beta_j(1-\omega_j) + \frac{k_j}{\phi+\rho})$ *for simplicity*.

3.2. Equilibrium under scenario D

Under this scenario, the developed region i may offer dynamic PES for encouraging the developing region j to control air pollution. Similarly to the previous study (Jiang et al., 2019), the payment depends on the actual amounts of reduced air pollutants $\alpha_j(t)e_j(t)$ in the region j. Assume that region i is willing to pay $\varepsilon(t)$ for per-unit reduced air pollutants. After receiving payments from region i, region j will share the same unit cost to reduce air pollutants, i.e., $c_i = c_j = c - \delta$ under this scenario. Thus, this game consists of an additional control variable $\varepsilon(t)$ compared with the model under the scenario N. They maximize their own gross discounted instantaneous payoffs, respectively, as follows:

$$\begin{aligned}\Pi_i^D = \max_{e_i(t),\alpha_i(t),\varepsilon(t)} \int_0^\infty & e^{-\rho t}\Big\{A_i e_i(t) - \frac{1}{2}e_i^2(t) - \frac{1}{2}c_i[\alpha_i(t)e_i(t)]^2 - k_i P(t) \\ & + \beta_i\left[\bar{e}_i - (1-\omega_i)(1-\alpha_i(t))e_i(t) - \omega_j\left(1-\alpha_j(t)\right)e_j(t)\right] \\ & - \varepsilon(t)\alpha_j(t)e_j(t)\Big\}dt,\end{aligned} \tag{3.7}$$

$$\begin{aligned}\Pi_j^D = \max_{e_j(t),\alpha_j(t)} \int_0^\infty & e^{-\rho t}\Big\{A_j e_j(t) - \frac{1}{2}e_j^2(t) - \frac{1}{2}c_j\left[\alpha_j(t)e_j(t)\right]^2 - k_j P(t) \\ & + \beta_j\left[\bar{e}_j - \left(1-\omega_j\right)\left(1-\alpha_j(t)\right)e_j(t) - \omega_i(1-\alpha_i(t))e_i(t)\right] \\ & + \varepsilon(t)\alpha_j(t)e_j(t)\Big\}dt.\end{aligned} \tag{3.8}$$

Similarly, considering the associated HJB functions $V_h^D(P)$ as follows:

$$\begin{aligned}\rho V_i^D(P) = \max_{e_i(t),\,\alpha_i(t),\,\varepsilon(t)} & A_i e_i(t) - \frac{1}{2}e_i^2(t) - \frac{1}{2}c_i[\alpha_i(t)e_i(t)]^2 - k_i P(t) \\ & + \beta_i\left[\bar{e}_i - (1-\omega_i)(1-\alpha_i(t))e_i(t) - \omega_j\left(1-\alpha_j(t)\right)e_j(t)\right] - \varepsilon(t)\alpha_j(t)e_j(t) \\ & + V_i^{D'}(P)\left\{[1-\alpha_i(t)]e_i(t) + \left[1-\alpha_j(t)\right]e_j(t) - \phi P(t)\right\} \\ & + V_i^{D''}(P)\frac{1}{2}(\sigma(P(t)))^2,\end{aligned} \tag{3.9}$$

$$\begin{aligned}\rho V_j^D(P) = \max_{e_j(t),\,\alpha_j(t)} & A_j e_j(t) - \frac{1}{2}e_j^2(t) - \frac{1}{2}c_j\left[\alpha_j(t)e_j(t)\right]^2 - k_j P(t) \\ & + \beta_j\left[\bar{e}_j - \left(1-\omega_j\right)\left(1-\alpha_j(t)\right)e_j(t) - \omega_i(1-\alpha_i(t))e_i(t)\right] + \varepsilon(t)\alpha_j(t)e_j(t) \\ & + V_j^{D'}(P)\left\{[1-\alpha_i(t)]e_i(t) + \left[1-\alpha_j(t)\right]e_j(t) - \phi P(t)\right\} \\ & + V_j^{D''}(P)\frac{1}{2}(\sigma(P(t)))^2,\end{aligned} \tag{3.10}$$

where $V_h^{D'}(P) = \frac{dV_h^D(P)}{dP}$ and $V_h^{D''}(P) = \frac{d^2V_h^D(P)}{dP^2}$.

By solving above HJB equations, the optimal feedback strategies for two regions can be obtained in Proposition 3.

Proposition 3. *Under the scenario D, the Markov-perfect Nash equilibrium solutions of the instantaneous emissions strategies $e_h(t)$, the proportion of emission* *reduction $\alpha_h(t)$, the dynamic payment for per-unit reduced air pollutants $\varepsilon(t)$ and the total payoff of region h, $h = i, j$ can be written as follows*:

$$
\begin{aligned}
e_i^{D*} &= A_i-\beta_i(1-\omega_i)-\frac{k_i}{\phi+\rho},\quad e_j^{D*} = A_j-\beta_j(1-\omega_j)-\frac{k_j}{\phi+\rho},\\
\alpha_i^{D*} &= \frac{\beta_i(1-\omega_i)(\phi+\rho)+k_i}{c_i[A_i(\phi+\rho)-\beta_i(1-\omega_i)(\phi+\rho)-k_i]},\\
\alpha_j^{D*} &= \frac{\left[\beta_i\omega_j+\beta_j(1-\omega_j)\right](\phi+\rho)+k_i+k_j}{2c_j\left[A_j(\phi+\rho)-\beta_j(1-\omega_j)(\phi+\rho)-k_j\right]},\\
\varepsilon(t)^* &= \frac{\left[\beta_i\omega_j-\beta_j(1-\omega_j)\right](\phi+\rho)-k_j+k_i}{2(\phi+\rho)},\\
V_i^{D*} &= -\frac{k_i}{\phi+\rho}P(t)+\frac{1}{\rho}\left\{\left(1+\frac{1}{c_j}\right)\frac{k_ik_j}{(\phi+\rho)^2}+\frac{1}{2}\left(1+\frac{1}{c_i}\right)\frac{k_i^2}{(\phi+\rho)^2}\right.\\
&\quad+\left[\beta_i\omega_j\left(1+\frac{1}{c_j}\right)-\frac{\varepsilon(t)^*}{c_j}\right]\frac{k_j}{\phi+\rho}-\left\{A_i+A_j-\beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right)\right.\\
&\quad\left.-\beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)-\frac{\varepsilon(t)^*}{c_j}\right\}\frac{k_i}{\phi+\rho}+\frac{1}{2}\beta_i^2(1-\omega_i)^2\left(1+\frac{1}{c_i}\right)\\
&\quad+\beta_i(\overline{e}_i-A_i(1-\omega_i)-A_j\omega_j)+\frac{1}{2}A_i^2+\left(1+\frac{1}{c_j}\right)\beta_i\beta_j\omega_j(1-\omega_j)\\
&\quad\left.-\frac{(\varepsilon(t)^*)^2}{c_j}+\frac{\left[\beta_i\omega_j-\beta_j(1-\omega_j)\right]\varepsilon(t)^*}{c_j}\right\},\\
V_j^{D*} &= -\frac{k_j}{\phi+\rho}P(t)\frac{1}{\rho}\left\{\left(1+\frac{1}{c_i}\right)\frac{k_ik_j}{(\phi+\rho)^2}+\frac{1}{2}\left(1+\frac{1}{c_j}\right)\frac{k_j^2}{(\phi+\rho)^2}\right.\\
&\quad+\beta_j\omega_i\left(1+\frac{1}{c_i}\right)\frac{k_i}{\phi+\rho}-\left\{A_i+A_j-\beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right)\right.\\
&\quad\left.-\beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)-\frac{\varepsilon(t)^*}{c_j}\right\}\frac{k_j}{\phi+\rho}+\frac{1}{2}\beta_j^2(1-\omega_j)^2\left(1+\frac{1}{c_j}\right)\\
&\quad+\beta_j(\overline{e}_j-A_j(1-\omega_j)-A_i\omega_i)+\frac{1}{2}A_j^2+\left(1+\frac{1}{c_i}\right)\beta_i\beta_j\omega_i(1-\omega_i)\\
&\quad\left.+\frac{(\varepsilon(t)^*)^2}{2c_j}+\frac{\beta_j(1-\omega_j)\varepsilon(t)^*}{c_j}\right\}.
\end{aligned}
\tag{3.11}
$$

Similarly to the scenario N, we calculate the variance and expectation of $P^D(t)$ under equilibrium states in Proposition 4.

Proposition 4. *Under scenario D, the expectation and variance of air pollution stock $P^D(t)$ satisfy*:

$$
\begin{cases}
E\left(P^D(t)\right)=\dfrac{\Omega^D}{\phi}+\left(P_0-\dfrac{\Omega^D}{\phi}\right)e^{-\phi t},\\[2ex]
S\left(P^D(t)\right)=\dfrac{\sigma^2\left\{\Omega^D-2\left[\Omega^D-\phi P_0\right]e^{-t\phi}+\left[\Omega^D-2\phi P_0\right]e^{-2\phi t}\right\}}{2\phi^2},
\end{cases}
\tag{3.12}
$$

where
$\Omega^D = A_i+A_j-(1+\frac{1}{c_i})(\beta_i(1-\omega_i)+\frac{k_i}{\phi+\rho})-\beta_j(1-\omega_j)-\frac{k_j}{\phi+\rho}-\frac{1}{2c_j}[\beta_i\omega_j+\beta_j(1-\omega_j)+\frac{k_i+k_j}{\phi+\rho}]$
for simplicity.

3.3. Equilibrium under scenario F

Under this scenario, the developed region i may offer a fixed-fee payment M to region j at the beginning of the plan. Different from the scenario D, region j will receive the payment only if it agrees to cooperate with region i. After receiving payments from region i, region j will share the same unit cost to reduce air pollutants, i.e., $c_i = c_j = c - \delta$ under this scenario. They maximize their own gross discounted instantaneous payoffs, respectively, as follows:

$$
\begin{aligned}
\Pi_i^F = \max_{e_i(t),\,\alpha_i(t)}\int_0^{\infty}e^{-\rho t}&\left\{A_ie_i(t)-\frac{1}{2}e_i^2(t)-\frac{1}{2}c_i[\alpha_i(t)e_i(t)]^2-k_iP(t)\right.\\
&\left.+\beta_i\left[\overline{e}_i-(1-\omega_i)(1-\alpha_i(t))e_i(t)-\omega_j\left(1-\alpha_j(t)\right)e_j(t)\right]\right\}dt-M,
\end{aligned}
\tag{3.13}
$$

$$
\begin{aligned}
\Pi_j^F = \max_{e_j(t),\,\alpha_j(t)}\int_0^{\infty}e^{-\rho t}&\left\{A_je_j(t)-\frac{1}{2}e_j^2(t)-\frac{1}{2}c_j\left[\alpha_j(t)e_j(t)\right]^2-k_jP(t)\right.\\
&\left.+\beta_j\left[\overline{e}_j-\left(1-\omega_j\right)\left(1-\alpha_j(t)\right)e_j(t)-\omega_i(1-\alpha_i(t))e_i(t)\right]\right\}dt+M.
\end{aligned}
\tag{3.14}
$$

Similarly, considering the associated HJB functions $V_h^F(P)$ as follows:

$$
\begin{aligned}
\rho V_i^F(P) = \max_{e_i(t),\,\alpha_i(t),\,\varepsilon(t)}&A_ie_i(t)-\frac{1}{2}e_i^2(t)-\frac{1}{2}c_i[\alpha_i(t)e_i(t)]^2-k_iP(t)\\
&+\beta_i\left[\overline{e}_i-(1-\omega_i)(1-\alpha_i(t))e_i(t)-\omega_j\left(1-\alpha_j(t)\right)e_j(t)\right]-M\\
&+V_i^{F'}(P)\left\{[1-\alpha_i(t)]e_i(t)+\left[1-\alpha_j(t)\right]e_j(t)-\phi P(t)\right\}\\
&+V_i^{F''}(P)\frac{1}{2}(\sigma(P(t)))^2,
\end{aligned}
\tag{3.15}
$$

$$
\begin{aligned}
\rho V_j^F(P) = \max_{e_j(t),\,\alpha_j(t)}&A_je_j(t)-\frac{1}{2}e_j^2(t)-\frac{1}{2}c_j\left[\alpha_j(t)e_j(t)\right]^2-k_jP(t)\\
&+\beta_j\left[\overline{e}_j-\left(1-\omega_j\right)\left(1-\alpha_j(t)\right)e_j(t)-\omega_i(1-\alpha_i(t))e_i(t)\right]+M\\
&+V_j^{F'}(P)\left\{[1-\alpha_i(t)]e_i(t)+\left[1-\alpha_j(t)\right]e_j(t)-\phi P(t)\right\}\\
&+V_j^{F''}(P)\frac{1}{2}(\sigma(P(t)))^2,
\end{aligned}
\tag{3.16}
$$

where $V_h^{F'}(P)=\frac{dV_h^F(P)}{dP}$ and $V_h^{F''}(P)=\frac{d^2V_h^F(P)}{dP^2}$.

By solving above HJB equations, the optimal feedback strategies for two regions can be obtained in Proposition 5.

Proposition 5. *Under the scenario F, given a fixed-fee payment M, the Markov-perfect Nash equilibrium solutions of the instantaneous emissions strategies $e_h(t)$, the proportion of emission reduction $\alpha_h(t)$ and the total payoff of region h, $h = i, j$ can be written as follows*:

$$
\begin{aligned}
e_i^{F*} &= A_i-\beta_i(1-\omega_i)-\frac{k_i}{\phi+\rho},\quad e_j^{F*} = A_j-\beta_j(1-\omega_j)-\frac{k_j}{\phi+\rho},\\
\alpha_i^{F*} &= \frac{\beta_i(1-\omega_i)(\phi+\rho)+k_i}{c_i[A_i(\phi+\rho)-\beta_i(1-\omega_i)(\phi+\rho)-k_i]},\\
\alpha_j^{F*} &= \frac{\beta_j(1-\omega_j)(\phi+\rho)+k_j}{c_j\left[A_j(\phi+\rho)-\beta_j(1-\omega_j)(\phi+\rho)-k_j\right]},\\
V_i^{F*} &= -\frac{k_i}{\phi+\rho}P(t)+\frac{1}{\rho}\left\{\left(1+\frac{1}{c_j}\right)\frac{k_ik_j}{(\phi+\rho)^2}+\frac{1}{2}\left(1+\frac{1}{c_i}\right)\frac{k_i^2}{(\phi+\rho)^2}\right.\\
&\quad+\beta_i\omega_j\left(1+\frac{1}{c_j}\right)\frac{k_j}{\phi+\rho}-\left\{A_i+A_j-\beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right)\right.\\
&\quad\left.-\beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)\right\}\frac{k_i}{\phi+\rho}+\frac{1}{2}\beta_i^2(1-\omega_i)^2\left(1+\frac{1}{c_i}\right)\\
&\quad\left.+\beta_i(\overline{e}_i-A_i(1-\omega_i)-A_j\omega_j)+\frac{1}{2}A_i^2+\left(1+\frac{1}{c_j}\right)\beta_i\beta_j\omega_j(1-\omega_j)-M\right\},\\
V_j^{F*} &= -\frac{k_j}{\phi+\rho}P(t)+\frac{1}{\rho}\left\{\left(1+\frac{1}{c_i}\right)\frac{k_ik_j}{(\phi+\rho)^2}+\frac{1}{2}\left(1+\frac{1}{c_j}\right)\frac{k_j^2}{(\phi+\rho)^2}\right.\\
&\quad+\beta_j\omega_i\left(1+\frac{1}{c_i}\right)\frac{k_i}{\phi+\rho}-\left\{A_i+A_j-\beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right)\right.\\
&\quad\left.-\beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)\right\}\frac{k_j}{\phi+\rho}+\frac{1}{2}\beta_j^2(1-\omega_j)^2\left(1+\frac{1}{c_j}\right)\\
&\quad\left.+\beta_j(\overline{e}_j-A_j(1-\omega_j)-A_i\omega_i)+\frac{1}{2}A_j^2+\left(1+\frac{1}{c_i}\right)\beta_i\beta_j\omega_i(1-\omega_i)+M\right\}.
\end{aligned}
\tag{3.17}
$$

Similarly to the scenario N, we calculate the variance and expectation of $P^D(t)$ under equilibrium states in Proposition 6.

Proposition 6. *Under the scenario F, the expectation and variance of air pollution stock $P^N(t)$ satisfy:*

$$\begin{cases} E\left(P^F(t)\right) = \dfrac{\Omega^F}{\phi} + \left(P_0 - \dfrac{\Omega^F}{\phi}\right)e^{-\phi t}, \\ S\left(P^F(t)\right) = \dfrac{\sigma^2\left\{\Omega^F - 2\left[\Omega^F - \phi P_0\right]e^{-t\phi} + \left[\Omega^F - 2\phi P_0\right]e^{-2\phi t}\right\}}{2\phi^2}, \end{cases} \tag{3.18}$$

where $\Omega^F = A_i + A_j - (1+\frac{1}{c_i})(\beta_i(1-\omega_i) + \frac{k_i}{\phi+\rho}) - (1+\frac{1}{c_j})(\beta_j(1-\omega_j) + \frac{k_j}{\phi+\rho})$ *for simplicity.*

4. Numerical illustration

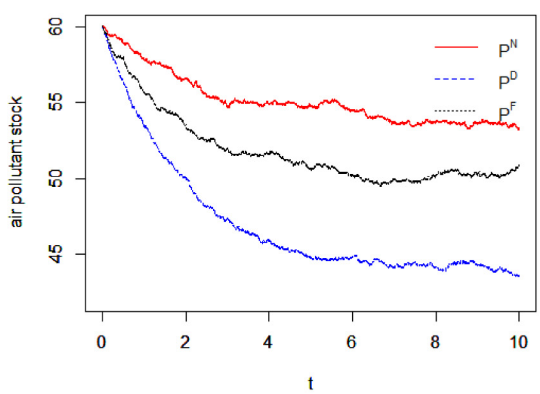
The main purpose of our numerical analysis is to compare the optimal strategies of both regions under the three PES scenarios. By referring to previous studies (Chang et al., 2018; Yeung and Petrosyan, 2008), the base parameters are set as follows to ensure the asymmetry of development level between regions i and j: $A = 30$, $\theta = 0.85$, so $A_i = 30$ and $A_j = \theta A = 25.5$; $\bar{e}_i = 40$ and $\bar{e}_j = 30$; $k = 3$, $\mu = \frac{1}{3}$, so $k_i = 3$ and $k_j = \mu k = 1$; $\beta_i = 6$ and $\beta_j = 3$; $\omega_i = 0.2$ and $\omega_j = 0.6$. The unit cost of emission reduction is set as $c = 2$ and $\delta = 1.2$. So we have $c_i = c - \delta = 0.8$ and $c_j = c = 2$ under the scenario N, and $c_i = c_j = c - \delta = 0.8$ under the scenario D and F. Then we set the natural degradation rate $\phi = 0.55$, the discount rate $\rho = 0.08$, the volatility rate $\sigma = 0.07$, the fixed-fee payment $M = 12$ and the initial amounts of air pollutants $P_0 = 60$. It should be noted that the parameter setting is not unique. This paper focuses on the relationship among different parameters rather than specific values.

4.1. Analysis of equilibrium trajectories

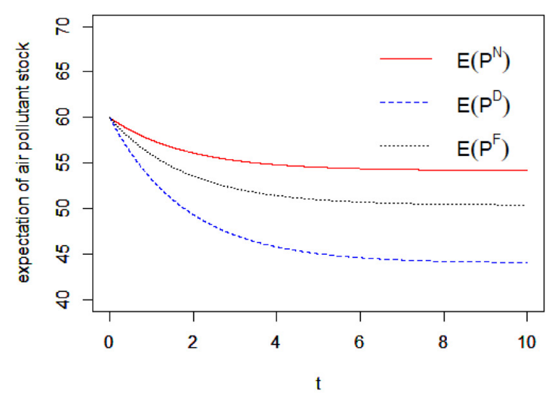
Here we compare the trajectories of the state variable $P(t)$ in three scenarios. Since the stochastic differential equation of the state variable cannot be solved analytically, we may characterize its evolution path by simulations (Prasad and Sethi, 2004). According to Eq. (2.4), the stochastic differential equations of $P(t)$ in three scenarios can be written in discrete forms:

$$P(t+\Delta) = P(t) + (\Omega^s - \phi P(t))\Delta + \sigma\sqrt{P(t)}\sqrt{\Delta}\xi(t), \tag{4.1}$$

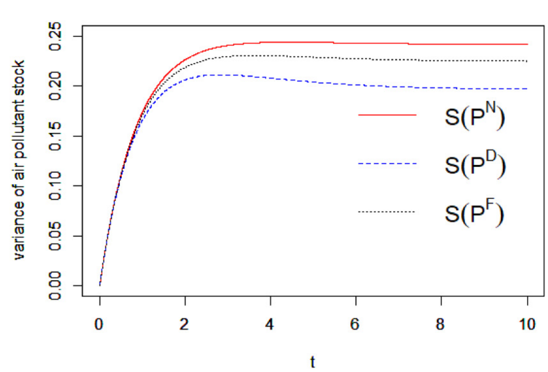
where $\xi(t) \sim N(0,1)$ are independent and identically distributed (i.i.d.) random variables and $s \in \{N, D, F\}$. The tiny time step is set as $\Delta = 0.001$. Then



(a) A simulated path of air pollutant stocks



(b) Expectation of air pollutant stocks



(c) Variance of air pollutant stocks

Fig. 1. The evolution path of air pollutant stocks.

the evolution path of $P(t)$ can be simulated by R language in Fig. 1(a). Then Fig. 1(b) and (c) illustrate the expectation and variance of $P(t)$ in three scenarios.

Fig. 1 demonstrates that the amounts of air pollutants under the scenario D has the lowest expectation and variance in our parameter settings. The second is that in the scenario F. The amounts of air pollutants under the scenario N has the highest expectation and variance. Formally, we have

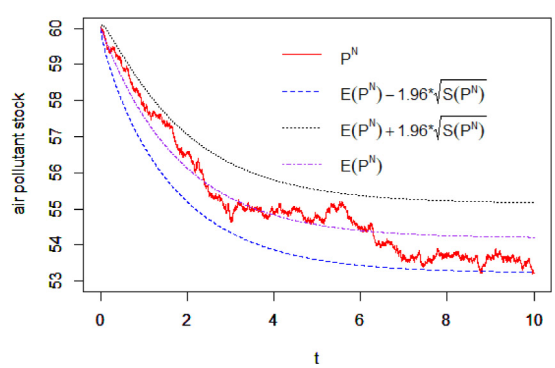
$$\begin{cases} E\left(P^D(t)\right) < E\left(P^F(t)\right) < E\left(P^N(t)\right), \\ S\left(P^D(t)\right) < S\left(P^F(t)\right) < S\left(P^N(t)\right). \end{cases} \tag{4.2}$$

These results are reasonable and conform to reality. Under the scenario N, two regions only consider their own profits and do not cooperate with each other. The developing region j cannot share the lower unit cost for emission reduction. Thus, the air pollution problem is the severest. Under the scenario F, region i offers a fixed-fee payment to region j. It encourages the developing region update their emission systems and then region j could share the same unit cost for emission reduction with the developed region i. This mechanism helps to control the air pollution, but they do not make further cooperation on the specific quantity of emission reduction. Under the scenario D, the payment that region i offers to region j dynamically depends on the quantity of emission reduction. It stimulates region j to reduce more emission to obtain more compensations. Thus, the control of air pollution is the most efficient under the scenario D.

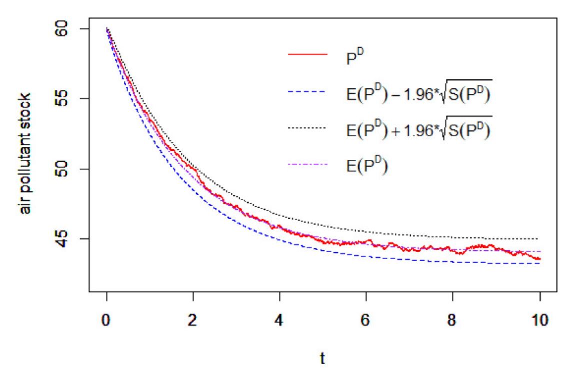
Then the confidence interval theory is used to estimate the variation range of $P(t)$. There are two types of estimation in statistics: point estimation and interval estimation. Compared with point estimation, interval estimation improves the reliability of the estimation results (Neyman, 1937). Therefore, we calculate the confidence interval of $P(t)$ according to its expectation and variance at a 95% significant level (Zwillinger, 1998). The intervals are computed as $\left[E(P(t)) - 1.96\sqrt{S(P(t))}, E(P(t)) + 1.96\sqrt{S(P(t))}\right]$ and presented in Fig. 2. Fig. 2 shows that, when benchmarked to the point estimation $E(P(t))$, the trajectories of $P(t)$ fluctuate within the interval $\left[E(P(t)) - 1.96\sqrt{S(P(t))}, E(P(t)) + 1.96\sqrt{S(P(t))}\right]$. It demonstrates the prediction accuracy of interval estimation. Hence, the confidence interval provides a powerful predictive diagnosis model for policy makers in the JPCAP area. It includes stochastic effects in the models, which are often ignored in the past.

4.2. Sensitivity analysis

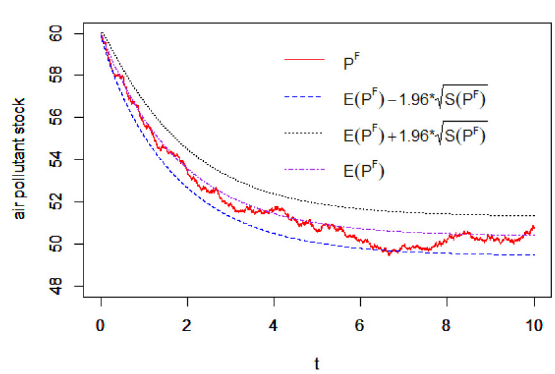
Here we perform the sensitivity analysis for the subjective parameters in our model. Since other parameters are imposed exclusively to guarantee the asymmetry and the initial state of regions, we only focus on the changes of keys parameters in the equilibrium strategies, including β_i, β_j, ω_i, ω_j, c, δ, k_i,



(a) Scenario N



(b) Scenario D



(c) Scenario F

Fig. 2. The confidence interval of air pollutant stocks under three scenarios.

k_j, ϕ and ρ. Based on the initial values set at the beginning of Section 4, these parameters are varied by −30%, −15%, 0, +15% and +30% of initial values, respectively. One parameter is varied while the other parameters remain unchanged (Vardar and Zaccour, 2018; Pnevmatikos et al., 2018). By varying these parameters within a range of ±30%, we demonstrate the robustness of our model to parameter settings. The impacts on decision variables and payoff functions due to the changes in parameters are presented in Table 1.

4.2.1. Changes in β_h

The larger utility coefficient of air quality β_h encourages less air pollution emission and more emission reduction in region h. It increases the utility of air quality and decreases the damage costs due to fewer air pollutants. Thus, it has a positive effect on the total payoff V_h. The other region is also beneficial from the fewer damage costs, so the total payoff increase with β_h. Such changes on environmental preference have a positive tendency to increase the investment of PES, since the developed region is willing to support the developing region for better environment. In addition, under the scenario D, the increase of β_i not only stimulates the developed region to invest more in ε^*, but also prompts the developing region to reduce more emissions α_j^D. Therefore, compared with the fixed-fee PES strategy, the dynamic PES strategy can generate more enthusiasm of the backward region to control air pollution and attain a win-win situation.

4.2.2. Changes in ω_h

The larger diffusion coefficient ω_h means that more air pollutants will be transmitted into the other region. It benefits the own air quality and economy in region h but harms that in the other region. With larger diffusion coefficient ω_h, the region h is encouraged to make more production activities and thus generates more air emission and reduces less emissions. Under the situation D, the increase of ω_j raises more PES investment ε^* in the JPCAP region. Conversely, the increase of ω_i has no impact on the investment ε^*. It demonstrates that the developed region i is less affected by the transregional air pollution problem.

4.2.3. Changes in c and δ

The increase in the unit cost of emission reduction c does not affect air pollution emission directly, yet it discourages the emission reduction α_h. Besides, the increases in costs surely has a negative effect on the total payoff V_h. Conversely, an increase in δ raises the total payoff of both sides in all three scenarios. The higher δ means the lower cost of capital in finance. It implies that the administration may take some actions to reduce the cost of emission reduction, e.g., by providing preferential interest rate financing for emission reduction projects.

4.2.4. Changes in k_h

The higher unit degradation cost k_h increases the damage cost in region h and has a negative effect on the payoff. It encourages less air emission and more emission reduction. Then less air pollution will improve air quality and decrease the damage cost in the other region. Thus, it benefits the total payoff in the other region. For the developed region i, it is willing to support more investment to the developing region j for air pollution control with the increasing of k_i. Furthermore, under the scenarios N and F, an increase in k_i has no direct impact on the emission reduction in the developing region. But under the scenario D, the developing region increases the investment in local emission reduction α_h^D with the increase of k_i. It indicates that the dynamic PES strategy can encourage the backward region to cut more emissions.

4.2.5. Changes in ϕ

The increase in the natural degradation rate ϕ stimulates more emissions e_h and it brings more benefits V_h. Since the environment is more capable of self-purification, the emission reduction α_h and the ecological compensation amounts ε^* appear to be less important. Thus, for the larger natural degradation rate ϕ, more efforts to improve the self-purification of the environment are required, such as tree planting and groundwater monitoring (Newell et al., 2021).

4.2.6. Changes in ρ

The discount rate ρ describes the time preference of two regions. The larger discount rate ρ means the regions will pay more attention to current production and consumption rather than future payoff. It stimulates two regions to increase production emissions to obtain more economic benefits while ignoring the investment in emission reduction and ecological compensation, which is consistent with the past development mode of "high consumption, high pollution, and high growth" in China. However, this mode is unsustainable due to the constraints of resource carrying capacity. Thus, it is necessary to focus on the sustainable development of the economy and the environment.

5. Conclusions

In this paper, we consider the transboundary air pollution control problem between two asymmetric regions within the JPCAP region. The stochastic differential game model is employed to model the diffusion of air

Table 1
Sensitivity analysis: the impacts on decision variables and payoff functions due to the changes of parameters.

Parm	e_i^N	e_i^D	e_i^F	e_j^N	e_j^D	e_j^F	α_i^N	α_i^D	α_i^F	α_j^N	α_j^D	α_j^F	ε
$\beta_i(4.2 \to 7.8)$	−	−	−	×	×	×	+	+	+	×	+	×	+
$\beta_j(2.1 \to 3.9)$	×	×	×	−	−	−	×	×	×	+	+	+	+
$\omega_i(0.14 \to 0.26)$	+	+	+	×	×	×	−	−	−	×	×	×	×
$\omega_j(0.42 \to 0.78)$	×	×	×	+	+	+	×	×	×	−	+	−	+
$c(1.4 \to 2.6)$	×	×	×	×	×	×	−	−	−	−	−	−	×
$\delta(0.84 \to 1.56)$	×	×	×	×	×	×	+	+	+	×	+	+	×
$k_i(2.1 \to 3.9)$	−	−	−	×	×	×	+	+	+	×	+	×	+
$k_j(0.7 \to 1.3)$	×	×	×	−	−	−	×	×	×	+	+	+	−
$\phi(0.385 \to 0.715)$	+	+	+	+	+	+	−	−	−	−	−	−	−
$\rho(0.056 \to 0.104)$	+	+	+	+	+	+	−	−	−	−	−	−	−

Parm	V_i^N	V_i^D	V_i^F	V_j^N	V_j^D	V_j^F
$\beta_i(4.2 \to 7.8)$	+	+	+	+	+	+
$\beta_j(2.1 \to 3.9)$	+	+	+	+	+	+
$\omega_i(0.14 \to 0.26)$	+	+	+	−	−	−
$\omega_j(0.42 \to 0.78)$	−	−	−	+	+	+
$c(1.4 \to 2.6)$	−	−	−	−	−	−
$\delta(0.84 \to 1.56)$	+	+	+	+	+	+
$k_i(2.1 \to 3.9)$	−	−	−	+	+	+
$k_j(0.7 \to 1.3)$	+	+	+	−	−	−
$\phi(0.385 \to 0.715)$	+	+	+	+	+	+
$\rho(0.056 \to 0.104)$	−	−	−	−	−	−

pollution. We derive the optimal feedback Nash equilibrium of both regions in three PES scenarios by using the HJB equation. Through numerical simulations, we demonstrate that both dynamic and fixed-fee PES strategies are effective at reducing air pollution stocks when compared to the no PES scenario. Furthermore, the dynamic PES strategy outperforms the fixed-fee PES strategy, which motivates the developing region to reduce more emissions. In addition, the confidence interval theory is used to estimate the variation range of air pollution stocks, which provides a powerful diagnostic tool for policy-makers.

This study still has several limitations that call for further research. The stochastic differential game theory is used to characterize the uncertainty in air pollution control, but it only works in one dimension. Other factors, such as the spatial dimension, can also be considered. Â From a practical perspective, more than two members in JPCAP should be included in the model. Some other models and algorithms to handle the stochastic diffusion, e.g., spatiotemporal models (Fan et al., 2020), deep learning algorithms (Sammen et al., 2020; Banan et al., 2020), and multi-objective optimization (Wang et al., 2020; Khanali et al., 2021), will be explored in the future.

CRediT authorship contribution statement

Conceptualization and literature review, L. Xiao; methodology, C. Wang and J. Wang; calculation and simulation, J. Liu; writing—original draft, L. Xiao and J. Liu; writing—review& editing, C. Wang; All authors read and approved this version.

Data availability

Data will be made available on request.

Declaration of competing interest

The authors confirm that this work is original and has not been published elsewhere, nor it is currently under consideration for publication elsewhere. The authors declare no conflict of interest.

Acknowledgements

This research was funded by Six Talent Peaks Project in Jiangsu Province [JY-095], the National Natural Science Foundation of China [72174076], National Natural Science Foundation of China [11971202], Outstanding Young Foundation of Jiangsu Province [BK20200042] and a scientific grant by Jiangsu Commission of Health [ZD2021009].

Appendix A. Proofs of propositions

Proposition 1. *By differentiating the objective function in* Eq. (3.3) *with respect to $e_i(t)$ and $\alpha_i(t)$, the optimal strategies for region i can be obtained*:

$$\begin{cases} A_i - e_i(t) - c_i\alpha_i^2(t)e_i(t) + \left[V_i^{N'}(P) - \beta_i(1-\omega_i)\right](1-\alpha_i(t)) = 0, \\ -c_ie_i^2(t)\alpha_i(t) + \beta_i(1-\omega_i)e_i(t) - V_i^{N'}(P)e_i(t) = 0. \end{cases}$$
$$\Rightarrow \begin{cases} e_i^N(t)^* = A_i - \beta_i(1-\omega_i) + V_i^{N'}(P), \\ \alpha_i^N(t)^* = \dfrac{\beta_i(1-\omega_i) - V_i^{N'}(P)}{c_i\left[A_i - \beta_i(1-\omega_i) + V_i^{N'}(P)\right]}. \end{cases} \tag{A.1}$$

Similarly, by differentiating the objective function in Eq. (3.4) with respect to $e_j(t)$ and $\alpha_j(t)$, the optimal strategies for region j can be obtained:

$$\begin{cases} e_j^N(t)^* = A_j - \beta_j(1-\omega_j) + V_j^{N'}(P), \\ \alpha_j^N(t)^* = \dfrac{\beta_j(1-\omega_j) - V_j^{N'}(P)}{c_j\left[A_j - \beta_j(1-\omega_j) + V_j^{N'}(P)\right]}. \end{cases} \tag{A.2}$$

Then, by substituting $e_i^N(t)^*$, $\alpha_i^N(t)^*$, $e_j^N(t)^*$, $\alpha_j^N(t)^*$ back into Eqs. (3.3) and (3.4), we can yield:

$$\begin{aligned} \rho V_i^N(P) = & \left[-k_i - \phi V_i^{N'}(P) + \frac{\sigma^2}{2}V_i^{N''}(P)\right]P(t) \\ & + \left(1+\frac{1}{c_j}\right)V_i^{N'}(P)V_j^{N'}(P) + \frac{1}{2}\left(1+\frac{1}{c_i}\right)\left[V_i^{N'}(P)\right]^2 \\ & - \beta_i\omega_j\left(1+\frac{1}{c_j}\right)V_j^{N'}(P) + \left\{A_i + A_j - \beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right)\right. \\ & \left. - \beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)\right\}V_i^{N'}(P) + \frac{1}{2}\beta_i^2(1-\omega_i)^2\left(1+\frac{1}{c_i}\right) \\ & + \beta_i(\bar{e}_i - A_i(1-\omega_i) - A_j\omega_j) + \frac{1}{2}A_i^2 + \left(1+\frac{1}{c_j}\right)\beta_i\beta_j\omega_j(1-\omega_j), \end{aligned} \tag{A.3}$$

$$\begin{aligned} \rho V_j^N(P) = & \left[-k_j - \phi V_j^{N'}(P) + \frac{\sigma^2}{2}V_j^{N''}(P)\right]P(t) \\ & + \left(1+\frac{1}{c_i}\right)V_i^{N'}(P)V_j^{N'}(P) + \frac{1}{2}\left(1+\frac{1}{c_j}\right)\left[V_j^{N'}(P)\right]^2 \\ & - \beta_j\omega_i\left(1+\frac{1}{c_i}\right)V_i^{N'}(P) + \left\{A_i + A_j - \beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right)\right. \\ & \left. - \beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)\right\}V_j^{N'}(P) + \frac{1}{2}\beta_j^2(1-\omega_j)^2\left(1+\frac{1}{c_j}\right) \\ & + \beta_j(\bar{e}_j - A_j(1-\omega_j) - A_i\omega_i) + \frac{1}{2}A_j^2 + \left(1+\frac{1}{c_i}\right)\beta_i\beta_j\omega_i(1-\omega_i). \end{aligned} \tag{A.4}$$

Following the structure of Eqs. (A.3) and (A.4), the solution of differential equations can be written as a linear analytical formula:

$$\begin{cases} V_i^N(P) = m_1^NP + m_2^N, \\ V_j^N(P) = n_1^NP + n_2^N, \end{cases} \tag{A.5}$$

where m_1^N, m_2^N, n_1^N, n_2^N are constant independent with $P(t)$. Then $V_i^{N'}(P) = m_1^N$, $V_j^{N'}(P) = n_1^N$ and $V_i^{N''}(P) = V_j^{N''}(P) = 0$. Replacing m_1^N, m_2^N, n_1^N, n_2^N back into Eqs. (A.3) and (A.4), we can obtain:

$$\begin{aligned} & V_i^{N'}(P) = m_1^N = -\frac{k_i}{\phi+\rho}, \quad V_j^{N'}(P) = n_1^N = -\frac{k_j}{\phi+\rho}, \\ & m_2^N = \frac{1}{\rho}\left\{\left(1+\frac{1}{c_j}\right)\frac{k_ik_j}{(\phi+\rho)^2} + \frac{1}{2}\left(1+\frac{1}{c_i}\right)\frac{k_i^2}{(\phi+\rho)^2} + \beta_i\omega_j\left(1+\frac{1}{c_j}\right)\frac{k_j}{\phi+\rho}\right. \\ & \quad - \left\{A_i + A_j - \beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right) - \beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)\right\}\frac{k_i}{\phi+\rho} \\ & \quad + \frac{1}{2}\beta_i^2(1-\omega_i)^2\left(1+\frac{1}{c_i}\right) + \beta_i(\bar{e}_i - A_i(1-\omega_i) - A_j\omega_j) \\ & \quad \left. + \frac{1}{2}A_i^2 + \left(1+\frac{1}{c_j}\right)\beta_i\beta_j\omega_j(1-\omega_j)\right\}, \\ & n_2^N = \frac{1}{\rho}\left\{\left(1+\frac{1}{c_i}\right)\frac{k_ik_j}{(\phi+\rho)^2} + \frac{1}{2}\left(1+\frac{1}{c_j}\right)\frac{k_j^2}{(\phi+\rho)^2} + \beta_j\omega_i\left(1+\frac{1}{c_i}\right)\frac{k_i}{\phi+\rho}\right. \\ & \quad - \left\{A_i + A_j - \beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right) - \beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)\right\}\frac{k_j}{\phi+\rho} \\ & \quad + \frac{1}{2}\beta_j^2(1-\omega_j)^2\left(1+\frac{1}{c_j}\right) + \beta_j(\bar{e}_j - A_j(1-\omega_j) - A_i\omega_i) \\ & \quad \left. + \frac{1}{2}A_j^2 + \left(1+\frac{1}{c_i}\right)\beta_i\beta_j\omega_i(1-\omega_i)\right\}. \end{aligned} \tag{A.6}$$

Thus, the results in Proposition 1 are desired.

Proposition 2. *According to* Eq. (2.4)*, then we have*

$$\begin{cases} dP(t) = \{\Omega^N - \phi P(t)\}dt + \sigma\sqrt{P(t)}dr(t), \\ P(0) = P_0 \geq 0, \end{cases} \tag{A.7}$$

where $\Omega^N = (1-\alpha_i^N(t)^*)e_i^N(t)^* + (1-\alpha_j^N(t)^*)e_j^N(t)^*$ *for simplicity. With the optimal solutions in* Proposition 1*, we can calculate that*

$$\begin{aligned}\Omega^N = A_i + A_j - \left(1+\frac{1}{c_i}\right)\left(\beta_i(1-\omega_i) + \frac{k_i}{\phi+\rho}\right) \\ -\left(1+\frac{1}{c_j}\right)\left(\beta_j(1-\omega_j) + \frac{k_j}{\phi+\rho}\right).\end{aligned} \tag{A.8}$$

By taking Ito integral for both sides of Eq. (A.7), we have

$$\begin{cases} P(t) = P(0) + \int_0^t [\Omega^N - \phi P(s)]ds + \int_0^t \sigma\sqrt{P(s)}dr(s), \\ E(P(0)) = P_0. \end{cases} \tag{A.9}$$

Note that $P(t)$ and $r(t)$ are independent and $r(t)$ is a standard Brownian motion with mean 0. Then, we have

$$E(P(t)) = P_0 + \int_0^t [\Omega^N - \phi E(P(s))]ds. \tag{A.10}$$

Therefore, the stochastic differential equation is reduced to an ordinary differential equation with respect to $E[P(t)]$. The solution of Eq. (A.10) is

$$E(P(t)) = \frac{\Omega^N}{\phi} + \left(P_0 - \frac{\Omega^N}{\phi}\right)e^{-\phi t}. \tag{A.11}$$

Subsequently, for variance of $P(t)$, by applying the Ito Lemma to Eq. (A.7), we obtain:

$$\begin{cases} dP(t)^2 = \left\{(2\Omega^N + \sigma^2)P(t) - 2\phi P(t)^2\right\}dt + 2\sigma\sqrt{P(t)}P(t)dr(t), \\ P(0)^2 = P_0^2 \geq 0. \end{cases} \tag{A.12}$$

Then, by taking the similar steps mentioned above, the second moment $E(P(t)^2)$ can be calculated as

$$E\left(P(t)^2\right) = P_0^2 + \int_0^t \left\{(2\Omega^N + \sigma^2)E(P(s)) - 2\phi E\left(P(s)^2\right)\right\}ds. \tag{A.13}$$

By substituting the result in Eq. (A.11), it can be rewritten as the ordinary differential form

$$\begin{cases} \frac{dE\left(P(t)^2\right)}{dt} = (2\Omega^N + \sigma^2)\left(\frac{\Omega^N}{\phi} + \left(P_0 - \frac{\Omega^N}{\phi}\right)e^{-\phi t}\right) - 2\phi E\left(P(t)^2\right), \\ P(0)^2 = P_0^2 \geq 0. \end{cases} \tag{A.14}$$

Solving this linear non-homogeneous differential equation yields:

$$\begin{aligned} E\left(P(t)^2\right) = P_0^2 e^{-2\phi t} + \frac{\Omega^N(2\Omega^N + \sigma^2)}{2\phi^2}(1 - e^{-2\phi t}) \\ + \frac{(2\Omega^N + \sigma^2)(P_0\phi - \Omega^N)}{\phi^2}(e^{-\phi t} - e^{-2\phi t}). \end{aligned} \tag{A.15}$$

Note that the variance $S(P(t)) = E(P(t)^2) - (E(P(t)))^2$. Then we have

$$S(P(t)) = \frac{\sigma^2\left\{\Omega^N - 2[\Omega^N - \phi P_0]e^{-t\phi} + [\Omega^N - 2\phi P_0]e^{-2\phi t}\right\}}{2\phi^2}. \tag{A.16}$$

Thus, the results in Proposition 2 are desired.

Proposition 3. *Since here is a Stackelberg game between region i and j, we consider the optimal strategy for region j first. By differentiating the objective function in* Eq. (3.10) *with respect to* $e_j(t)$ *and* $\alpha_j(t)$*, the optimal strategies for region j can be obtained:*

$$\begin{aligned} &\begin{cases} A_j - e_j(t) - c_j\alpha_j^2(t)e_j(t) + \left[V_j^{D'}(P) - \beta_j(1-\omega_j)\right](1-\alpha_j(t)) + \varepsilon(t)\alpha_j(t) = 0, \\ -c_je_j^2(t)\alpha_j(t) + \beta_j(1-\omega_j)e_j(t) - V_j^{D'}(P)e_j(t) + \varepsilon(t)e_j(t) = 0. \end{cases} \\ \Rightarrow &\begin{cases} e_j^D(t)^* = A_j - \beta_j(1-\omega_j) + V_j^{D'}(P), \\ \alpha_j^D(t)^* = \frac{\beta_j(1-\omega_j) - V_j^{D'}(P) + \varepsilon(t)}{c_j\left[A_j - \beta_j(1-\omega_j) + V_j^{D'}(P)\right]}. \end{cases} \end{aligned} \tag{A.17}$$

Note that $de_j^D(t)^*/d\varepsilon(t) = 0$ and $d\alpha_j^D(t)^*/d\varepsilon(t) = 1/(c_je_j^{D*}(t))$. Then substituting $e_j^D(t)^*$, $\alpha_j^D(t)^*$ into Eq. (3.9) and differentiating the objective function with respect to $e_i(t)$, $\alpha_i(t)$ and $\varepsilon(t)$ yield:

$$\begin{aligned} &\begin{cases} A_i - e_i(t) - c_i\alpha_i^2(t)e_i(t) + \left[V_i^{D'}(P) - \beta_i(1-\omega_i)\right](1-\alpha_i(t)) = 0, \\ -c_ie_i^2(t)\alpha_i(t) + \beta_i(1-\omega_i)e_i(t) - V_i^{D'}(P)e_i(t) = 0, \\ \beta_i\omega_j - \left[\beta_j(1-\omega_j) - V_j^{D'}(P) + 2\varepsilon(t)\right] - V_i^{D'}(P) = 0. \end{cases} \\ \Rightarrow &\begin{cases} e_i^D(t)^* = A_i - \beta_i(1-\omega_i) + V_i^{D'}(P), \\ \alpha_i^D(t)^* = \frac{\beta_i(1-\omega_i) - V_i^{D'}(P)}{c_i\left[A_i - \beta_i(1-\omega_i) + V_i^{D'}(P)\right]}, \\ \varepsilon(t)^* = \frac{\beta_i\omega_j - \beta_j(1-\omega_j) + V_j^{D'}(P) - V_i^{D'}(P)}{2}. \end{cases} \end{aligned} \tag{A.18}$$

Then, by substituting $e_i^N(t)^*$, $\alpha_i^N(t)^*$, $e_j^N(t)^*$, $\alpha_j^N(t)^*$ back into Eqs. (3.9) and (3.10), we can yield:

$$\begin{aligned} \rho V_i^D(P) = &\left[-k_i - \phi V_i^{D'}(P) + \frac{\sigma^2}{2}V_i^{D''}(P)\right]P(t) + \left(1+\frac{1}{c_j}\right)V_i^{D'}(P)V_j^{D'}(P) \\ &+ \frac{1}{2}\left(1+\frac{1}{c_i}\right)\left[V_i^{D'}(P)\right]^2 + \left[-\beta_i\omega_j\left(1+\frac{1}{c_j}\right) + \frac{\varepsilon(t)^*}{c_j}\right]V_j^{D'}(P) \\ &+ \left[A_i + A_j - \beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right) - \beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right) - \frac{\varepsilon(t)^*}{c_j}\right]V_i^{D'}(P) \\ &+ \frac{1}{2}\beta_i^2(1-\omega_i)^2\left(1+\frac{1}{c_i}\right) + \beta_i(\bar{e}_i - A_i(1-\omega_i) - A_j\omega_j) + \frac{1}{2}A_i^2 \\ &+ \left(1+\frac{1}{c_j}\right)\beta_i\beta_j\omega_j(1-\omega_j) - \frac{(\varepsilon(t)^*)^2}{c_j} + \frac{\left[\beta_i\omega_j - \beta_j(1-\omega_j)\right]\varepsilon(t)^*}{c_j}, \end{aligned} \tag{A.19}$$

$$\begin{aligned}\rho V_j^D(P) = & \left[-k_j-\phi V_j^{D'}(P)+\frac{\sigma^2}{2}V_j^{D''}(P)\right]P(t)+\left(1+\frac{1}{c_i}\right)V_i^{D'}(P)V_j^{D'}(P)\\ & +\frac{1}{2}\left(1+\frac{1}{c_j}\right)\left[V_j^{D'}(P)\right]^2-\beta_j\omega_i\left(1+\frac{1}{c_i}\right)V_i^{D'}(P)\\ & +\left\{A_i+A_j-\beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right)-\beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)-\frac{\varepsilon(t)^*}{c_j}\right\}V_j^{D'}(P)\\ & +\frac{1}{2}\beta_j^2(1-\omega_j)^2\left(1+\frac{1}{c_j}\right)+\beta_j\left(\overline{e}_j-A_j(1-\omega_j)-A_i\omega_i\right)+\frac{1}{2}A_j^2\\ & +\left(1+\frac{1}{c_i}\right)\beta_i\beta_j\omega_i(1-\omega_i)+\frac{(\varepsilon(t)^*)^2}{2c_j}+\frac{\beta_j(1-\omega_j)\varepsilon(t)^*}{c_j}.\end{aligned} \tag{A.20}$$

Following the structure of Eqs. (A.19) and (A.20), the solution of differential equations can be written as a linear analytical formula:

$$\begin{cases} V_i^D(P)=m_1^DP+m_2^D,\\ V_j^D(P)=n_1^DP+n_2^D,\end{cases} \tag{A.21}$$

where $m_1^D, m_2^D, n_1^D, n_2^D$ are constant independent with $P(t)$. Then $V_i^{D'}(P)=m_1^D$, $V_j^{D'}(P)=n_1^D$ and $V_i^{D''}(P)=V_j^{D''}(P)=0$. Replacing $m_1^D, m_2^D, n_1^D, n_2^D$ back into Eqs. (A.19) and (A.20), we can obtain:

$$\begin{aligned} & V_i^{D'}(P)=m_1^D=-\frac{k_i}{\phi+\rho},\quad V_j^{D'}(P)=n_1^D=-\frac{k_j}{\phi+\rho},\\ & m_2^D=\frac{1}{\rho}\left\{\left(1+\frac{1}{c_j}\right)\frac{k_ik_j}{(\phi+\rho)^2}+\frac{1}{2}\left(1+\frac{1}{c_i}\right)\frac{k_i^2}{(\phi+\rho)^2}+\left[\beta_i\omega_j\left(1+\frac{1}{c_j}\right)-\frac{\varepsilon(t)^*}{c_j}\right]\frac{k_j}{\phi+\rho}\right.\\ & \quad-\left\{A_i+A_j-\beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right)-\beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)-\frac{\varepsilon(t)^*}{c_j}\right\}\frac{k_i}{\phi+\rho}\\ & \quad+\frac{1}{2}\beta_i^2(1-\omega_i)^2\left(1+\frac{1}{c_i}\right)+\beta_i\left(\overline{e}_i-A_i(1-\omega_i)-A_j\omega_j\right)+\frac{1}{2}A_i^2\\ & \quad\left.+\left(1+\frac{1}{c_j}\right)\beta_i\beta_j\omega_j(1-\omega_j)-\frac{(\varepsilon(t)^*)^2}{c_j}+\frac{\left[\beta_i\omega_j-\beta_j(1-\omega_j)\right]\varepsilon(t)^*}{c_j}\right\},\\ & n_2^D=\frac{1}{\rho}\left\{\left(1+\frac{1}{c_i}\right)\frac{k_ik_j}{(\phi+\rho)^2}+\frac{1}{2}\left(1+\frac{1}{c_j}\right)\frac{k_j^2}{(\phi+\rho)^2}+\beta_j\omega_i\left(1+\frac{1}{c_i}\right)\frac{k_i}{\phi+\rho}\right.\\ & \quad-\left\{A_i+A_j-\beta_i(1-\omega_i)\left(1+\frac{1}{c_i}\right)-\beta_j(1-\omega_j)\left(1+\frac{1}{c_j}\right)-\frac{\varepsilon(t)^*}{c_j}\right\}\frac{k_j}{\phi+\rho}\\ & \quad+\frac{1}{2}\beta_j^2(1-\omega_j)^2\left(1+\frac{1}{c_j}\right)+\beta_j\left(\overline{e}_j-A_j(1-\omega_j)-A_i\omega_i\right)+\frac{1}{2}A_j^2\\ & \quad\left.+\left(1+\frac{1}{c_i}\right)\beta_i\beta_j\omega_i(1-\omega_i)+\frac{(\varepsilon(t)^*)^2}{2c_j}+\frac{\beta_j(1-\omega_j)\varepsilon(t)^*}{c_j}\right\}.\end{aligned} \tag{A.22}$$

Thus, the results in Proposition 3 are desired.

Proposition 4. *According to* Eq. (2.4)*, then we have*

$$\begin{cases} dP(t)=\left\{\Omega^D-\phi P(t)\right\}dt+\sigma\sqrt{P(t)}dr(t),\\ P(0)=P_0\geq 0,\end{cases} \tag{A.23}$$

where $\Omega^D=(1-\alpha_i^D(t)^*)e_i^D(t)^*+(1-\alpha_j^D(t)^*)e_j^D(t)^*$ *for simplicity. With the optimal solutions in* Proposition 3*, we can calculate that*

$$\begin{aligned}\Omega^D = & A_i+A_j-\left(1+\frac{1}{c_i}\right)\left(\beta_i(1-\omega_i)+\frac{k_i}{\phi+\rho}\right)\\ & -\beta_j(1-\omega_j)-\frac{k_j}{\phi+\rho}-\frac{1}{2c_j}\left[\beta_i\omega_j+\beta_j(1-\omega_j)+\frac{k_i+k_j}{\phi+\rho}\right].\end{aligned} \tag{A.24}$$

Then the process is the same with that in Proposition 2.

Proposition 5. *The proof is similar with that in* Proposition 1. Omitted.

Proposition 6. *The proof is similar with that in* Proposition 2. Omitted.

References

Andrés-Domenech, P., Martn-Herrán, G., Zaccour, G., 2015. Cooperation for sustainable forest management: an empirical differential game approach. Ecol. Econ. 117, 118–128.

Angelevska, B., Atanasova, V., Andreevski, I., 2021. Urban air quality guidance based on measures categorization in road transport. Civ. Eng. J. 7 (2), 253–267.

Athanassoglou, S., Xepapadeas, A., 2012. Pollution control with uncertain stock dynamics: when, and how, to be precautious. J. Environ. Econ. Manag. 63 (3), 304–320.

Banan, A., Nasiri, A., Taheri-Garavand, A., 2020. Deep learning-based appearance features extraction for automated carp species identification. Aquac. Eng. 89, 102053.

Bellman, R., 1954. Dynamic programming and a new formalism in the calculus of variations. Proc. Natl. Acad. Sci. 40 (4), 231–235.

Bina, O., 2010. Environmental governance in China: weakness and potential from an environmental policy integration perspective. China Rev. 207–239.

Breton, M., Sbragia, L., Zaccour, G., 2010. A dynamic model for international environmental agreements. Environ. Resour. Econ. 45 (1), 25–48.

Chang, S., Wang, X., Wang, Z., 2015. Modeling and computation of transboundary industrial pollution with emission permits trading by stochastic differential game. PloS One 10 (9), e0138641.

Chang, S., Sethi, S.P., Wang, X., 2018. Optimal abatement and emission permit trading policies in a dynamic transboundary pollution game. Dyn. Games Applic. 8 (3), 542–572.

Claassen, R., Cattaneo, A., Johansson, R., 2008. Cost-effective design of agri-environmental payment programs: US experience in theory and practice. Ecol. Econ. 65 (4), 737–752.

De Frutos, J., Martín-Herrán, G., 2019. Spatial vs. non-spatial transboundary pollution control in a class of cooperative and non-cooperative dynamic games. Eur. J. Operat. Research 276 (1), 379–394.

Do, T.H., Vu, T.P., Catacutan, D., et al., 2018. Payment for forest environmental services in Vietnam: an analysis of buyers' perspectives and willingness. Ecosyst. Serv. 32, 134–143.

Dockery, D.W., Pope, C.A., Xu, X., Spengler, J.D., Ware, J.H., Fay, M.E., Ferris Jr., B.G., Speizer, F.E., 1993. An association between air pollution and mortality in six US cities. N. Engl. J. Med. 329 (24), 1753–1759.

Ekwueme, B.N., Agunwamba, J.C., 2021. Trend analysis and variability of air temperature and rainfall in regional river basins. Civ. Eng. J. 7 (5), 816–826.

Engel, S., Pagiola, S., Wunder, S., 2008. Designing payments for environmental services in theory and practice: an overview of the issues. Ecol. Econ. 65 (4), 663–674.

Fan, Y., Xu, K., Wu, H., Zheng, Y., Tao, B., 2020. Spatiotemporal modeling for nonlinear distributed thermal processes based on KL decomposition, MLP and LSTM network. IEEEAccess 8, 25111–25121.

Feistel, R., Hellmuth, O., 2021. Relative humidity: a control valve of the steam engine climate. J. Hum. Earth Future 2 (2), 140–182.

Huang, P., Zhang, X., Deng, X., 2006. Survey and analysis of public environmental awareness and performance in Ningbo, China: a case study on household electrical and electronic equipment. J. Clean. Prod. 14 (18), 1635–1643.

Iacus, S.M., 2008. Simulation And Inference for Stochastic Differential Equations: With R Examples. Vol. 486. Springer.

Jacobson, M.Z., 2009. Review of solutions to global warming, air pollution, and energy security. Energy Environ. Sci. 2 (2), 148–173.

Jiang, K., Merrill, R., You, D., Pan, P., Li, Z., 2019. Optimal control for transboundary pollution under ecological compensation: a stochastic differential game approach. J. Clean. Prod. 241, 118391.

Jørgensen, S., Zaccour, G., 2001. Time consistent side payments in a dynamic game of downstream pollution. J. Econ. Dyn. Control. 25 (12), 1973–1987.

Kamieniecki, S., Ferrall, M.R., 1991. Intergovernmental relations and clean-air policy in Southern California. Publius 21 (3), 143–154.

Kemajou-Brown, I., 2016. Brief history of optimal control theory and some recent developments. Probability on Algebraic And Geometric Structures. 668, p. 119.

Khanali, M., Akram, A., Behzadi, J., Mostashari-Rad, F., Saber, Z., Chau, K.-W., Nabavi-Pelesaraei, A., 2021. Multi-objective optimization of energy use and environmental emissions for walnut production using imperialist competitive algorithm. Appl. Energy 284, 116342.

Kolinjivadi, V., Adamowski, J., Kosoy, N., 2014. Recasting payments for ecosystem services (PES) in water resource management: a novel institutional approach. Ecosyst. Serv. 10, 144–154.

Lelieveld, J., Evans, J.S., Fnais, M., Giannadaki, D., Pozzer, A., 2015. The contribution of outdoor air pollution sources to premature mortality on a global scale. Nature 525 (7569), 367–371.

Li, S., 2014. A differential game of transboundary industrial pollution with emission permits trading. J. Optim. Theory Appl. 163 (2), 642–659.

Liu, M., Shadbegian, R., Zhang, B., 2017. Does environmental regulation affect labor demand in China? Evidence from the textile printing and dyeing industry. J. Environ. Econ. Manag. 86, 277–294.

Lyle, G., Bryan, B., Ostendorf, B., 2015. Identifying the spatial and temporal variability of economic opportunity costs to promote the adoption of alternative land uses in grain growing agricultural areas: an Australian example. J. Environ. Manag. 155, 123–135.

Mahanty, S., Suich, H., Tacconi, L., 2013. Access and benefits in payments for environmental services and implications for REDD+: lessons from seven PES schemes. Land Use Policy 31, 38–47.

Masoudi, N., Santugini, M., Zaccour, G., 2016. A dynamic game of emissions pollution with uncertainty and learning. Environ. Resour. Econ. 64 (3), 349–372.

Muñoz-Piña, C., Guevara, A., Torres, J.M., Braña, J., 2008. Paying for the hydrological services of Mexico's forests: analysis, negotiations and results. Ecol. Econ. 65 (4), 725–736.

Nakada, T., Takadama, K., Watanabe, S., 2013. Analysis of emission right prices in greenhouse gas emission trading via agent-based model. MultiagentGrid Syst. 9 (3), 227–246.

Newell, C.J., Adamson, D.T., Kulkarni, P.R., Nzeribe, B.N., Connor, J.A., Popovic, J., Stroo, H.F., 2021. Monitored natural attenuation to manage PFAS impacts to groundwater: scientific basis. Groundw. Monit. Remediat. 41 (4), 76–89.

Neyman, J., 1937. Outline of a theory of statistical estimation based on the classical theory of probability. Philos. Trans. R. Soc. London, Ser. A 236 (767), 333–380.

Pagiola, S., 2008. Payments for environmental services in Costa Rica. Ecol. Econ. 65 (4), 712–724.

Pnevmatikos, N., Vardar, B., Zaccour, G., 2018. When should a retailer invest in brand advertising? Eur. J. Oper. Res. 267 (2), 754–764.
Prasad, A., Sethi, S.P., 2004. Competitive advertising under uncertainty: a stochastic differential game approach. J. Optim. Theory Appl. 123 (1), 163–185.
Qin, M., Wang, X., Hu, Y., Huang, X., He, L., Zhong, L., Song, Y., Hu, M., Zhang, Y., 2015. Formation of particulate sulfate and nitrate over the Pearl River Delta in the fall: diagnostic analysis using the community multiscale air quality model. Atmos. Environ. 112, 81–89.
Roumasset, J., Wada, C.A., 2013. A dynamic approach to PES pricing and finance for interlinked ecosystem services: watershed conservation and groundwater management. Ecol. Econ. 87, 24–33.
Sammen, S.S., Ghorbani, M.A., Malik, A., Tikhamarine, Y., AmirRahmani, M., Al-Ansari, N., Chau, K.-W., 2020. Enhanced artificial neural network with Harris hawks optimization for predicting scour depth downstream of ski-jump spillway. Appl. Sci. 10 (15), 5160.
Sun, J., Dang, Z., Zheng, S., 2017. Development of payment standards for ecosystem services in the largest interbasin water transfer projects in the world. Agric. Water Manag. 182, 158–164.
Vardar, B., Zaccour, G., 2018. The strategic impact of adaptation in a transboundary pollution dynamic game. Environ. Model. Assess. 23 (6), 653–669.
Wang, X.H., 1998. Fee versus royalty licensing in a Cournot duopoly model. Econ. Lett. 60 (1), 55–62.
Wang, H., Zhao, L., 2018. A joint prevention and control mechanism for air pollution in the Beijing-Tianjin-Hebei region in China based on long-term and massive data mining of pollutant concentration. Atmos. Environ. 174, 25–42.
Wang, B., Hong, G., Qin, T., Fan, W.-R., Yuan, X.-C., 2019. Factors governing the willingness to pay for air pollution treatment: a case study in the Beijing-Tianjin-Hebei region. J. Clean. Prod. 235, 1304–1314.
Wang, W.-C., Xu, L., Chau, K.-W., Xu, D.-M., 2020. Yin-Yang firefly algorithm based on dimensionally Cauchy mutation. Expert Syst. Appl. 150, 113216.
Wunder, S., 2015. Revisiting the concept of payments for environmental services. Ecol. Econ. 117, 234–243.
Xiao, C., Chang, M., Guo, P., Gu, M., Li, Y., 2020. Analysis of air quality characteristics of Beijing-Tianjin-Hebei and its surrounding air pollution transport channel cities in China. J. Environ. Sci. 87, 213–227.
Xie, Y., Zhao, L., Xue, J., Gao, H.O., Li, H., Jiang, R., Qiu, X., Zhang, S., 2018. Methods for defining the scopes and priorities for joint prevention and control of air pollution regions based on data-mining technologies. J. Clean. Prod. 185, 912–921.
Xiong, C., Yang, D., Huo, J., Wang, G., 2017. Agricultural net carbon effect and agricultural carbon sink compensation mechanism in Hotan prefecture, China. Pol. J. Environ. Stud. 26 (1), 365–373.
Yang, W., Liu, W., Viña, A., Luo, J., He, G., Ouyang, Z., Zhang, H., Liu, J., 2013. Performance and prospects of payments for ecosystem services programs: evidence from China. J. Environ. Manag. 127, 86–95.
Ye, G., Zhao, J., 2016. Environmental regulation in a mixed economy. Environ. Resour. Econ. 65 (1), 273–295.
Yeung, D.W., 2007. Dynamically consistent cooperative solution in a differential game of transboundary industrial pollution. J. Optim. Theory Appl. 134 (1), 143–160.
Yeung, D.W., Petrosyan, L.A., 2008. A cooperative stochastic differential game of transboundary industrial pollution. Automatica 44 (6), 1532–1544.
Yi, Y., Xu, R., Zhang, S., 2017. A cooperative stochastic differential game of transboundary industrial pollution between two asymmetric nations. Math. Probl. Eng. 2017, 1–10.
Zhang, Q., Jiang, X., Tong, D., Davis, S.J., Zhao, H., Geng, G., Feng, T., Zheng, B., Lu, Z., Streets, D.G., et al., 2017. Transboundary health impacts of transported global air pollution and international trade. Nature 543 (7647), 705–709.
Zhao, N., Ghaemi, A., Wu, C., Band, S.S., Chau, K.-W., Zaguia, A., Mafarja, M., Mosavi, A.H., 2021. A decomposition and multi-objective evolutionary optimization model for suspended sediment load prediction in rivers. Eng. Applic. Comput. Fluid Mech. 15 (1), 1811–1829.
Zwillinger, D., 1998. Handbook of Differential Equations. Gulf Professional Publishing.